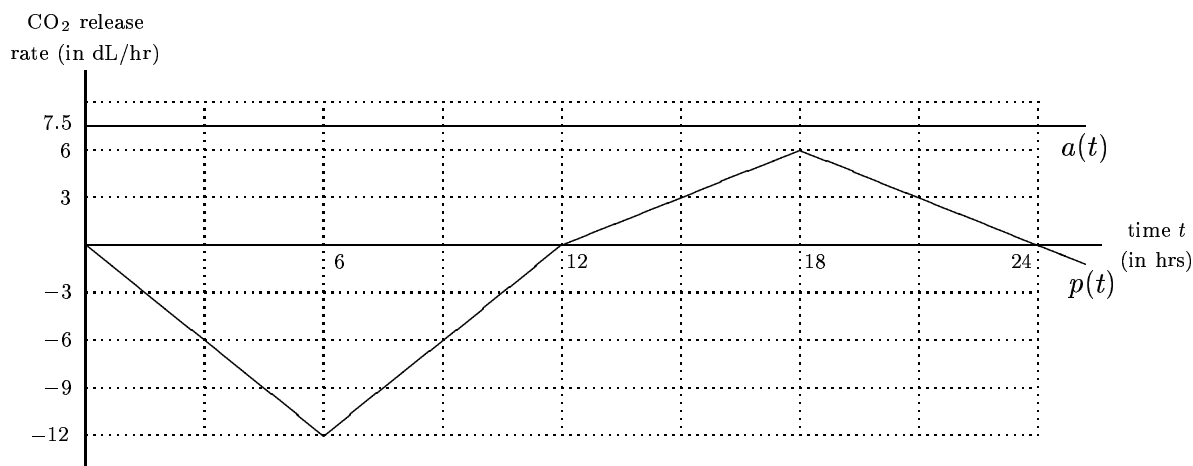


8. (14 points) A team of biologists seeking to develop alternate solutions to the use of pesticides proposes the following experiment. A plant infested by a large colony of aphids (small insects), is placed in a container originally saturated in dioxygen (O_2). They hope that the parasites can be suffocated by the CO_2 produced by their natural activities (e.g. breathing) and those of the plant (e.g. photosynthesis). Make the simplifying assumptions, which are approximations, that:

- (i) The container is sealed; no molecules enter or leave.
- (ii) The sum of the volumes of O_2 and CO_2 in the container is constant and equal to 117 deciliters (dL); i.e. 11.7 liters.
- (iii) At the start of the experiment ($t = 0$), there is no CO_2 in the container.

The light in the room is adjusted so as to mimic a “perfect” 24-hour period. The parasites produce CO_2 at a constant rate of 7.5 dL per hour. When the light is turned on, the plant absorbs CO_2 and releases O_2 . When the light is turned off, it does the opposite; i.e. uses O_2 and produces CO_2 .

The rates of release in the container of CO_2 (in dL per hour) for the parasites, $a(t)$, and the plant, $p(t)$, are plotted below. Note that negative rates correspond to absorption.



(a) By approximately how much does the volume $V(t)$ of CO_2 in the container change in the small time interval between t and $t + \Delta t$? Express your answer in terms of Δt , and of the rates $p(t)$ and $a(t)$.

Problem continued from previous page.

(b) Write an integral that gives the volume $V(12)$ of CO_2 in the container after 12 hours. Explain why your integral gives the value of $V(12)$. You will probably want to use the answer from **(a)** in your explanation.

(c) What is the value of $V(12)$? Explain how you obtained your answer.

ANSWER: $V(12) =$ _____.

(d) When the container becomes saturated in CO_2 , i.e. no O_2 remains, the parasites suffocate to death, and the experiment is stopped. Decide whether that will happen within the 24-hour period; and, if you think it will, estimate this time.