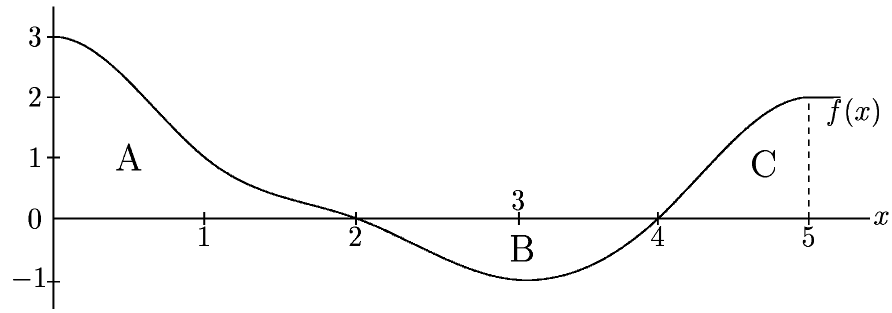
1. (15 pts.) For $0 \le x \le 5$, let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown in the figure. The areas of the regions bounded by the graph of f and the x-axis, and labeled A, B, C are equal to 2.5, 1, and 1, respectively.



(a) Find, as accurately as you can...

(i) the values of:

$g(2) = \int_0^2 f(x)dx = \mathbf{2.5}$, $g(4) = \int_0^4 f(x)dx = 2.5 - 1 = \mathbf{1.5}$, $g'(5) = f(5) = \mathbf{2}$.

(ii) the interval(s) on which g is decreasing.

From the FTC, we know $g'(x) = f(x)$. Since g is decreasing where g' is negative, we're looking for the intervals where f is negative. There's only one such interval: $\mathbf{2 < x < 4}$.

(iii) the interval(s) on which g is concave up.

Again, from the FTC, we know $g''(x) = f'(x)$. Since g is concave up where g'' is positive, we're looking for the intervals where f' is positive, i.e. the intervals on which f is increasing. From the graph of f, we see there's only one such interval: $\mathbf{3 < x < 5}$.

(iv) the value(s) of x and $g(x)$ for the value(s) of $0 \le x \le 5$ where $g(x)$ is largest.

We know g is increasing where its derivative, f, is positive, i.e. on $0 < x < 2$ and $4 < x < 5$, and decreasing on the interval $2 < x < 4$ where f is negative. Therefore, the largest value of g on the interval $0 \le x \le 4$ must occur at $x = 2$, where we have $g(2) = 2.5$. On the interval $4 \le x \le 5$, the largest value of g must occur at $x = 5$ where $g(5) = 2.5$. Therefore, g attains its maximum value of 2.5 at the two points, $x = 2$ and $x = 5$ of the interval.

Solution continued on next page.

Solution continued from previous page.

(b) Sketch as accurately as you can the graph of g. Make sure...

- that your graph is consistent with your answers to parts (a)-(d);
- to label any points on the graph where you know the coordinates of the point $(x, g(x))$.

There are only four points on the graph of g which are known exactly: $g(0) = 0$, $g(2) = 2.5$, $g(4) = 1.5$, and $g(5) = 2.5$.
We know g is increasing where its derivative, f, is positive, i.e. on $0 < x < 2$ and $4 < x < 5$. Similarly, g is decreasing on $2 < x < 4$. Also, as seen in the previous question, g has a local max at $x = 2$ and a local min at $x = 4$.
We know g is concave up where its second derivative, f', is positive, i.e. where f is increasing. Thus g is concave up on $3 < x < 5$. Similarly, g is concave down where f is decreasing, i.e. on $0 < x < 3$. Also, g has inflection points wherever f has a local min or a local max. From the graph, we estimate that happens for $x = 3$ only.
Compiling this information altogether produces a rough sketch of the graph of the function g (see below).