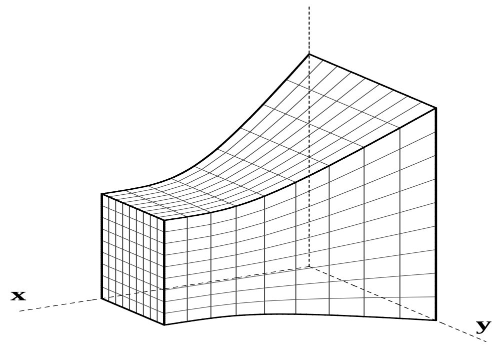
6. (15 pts.) Consider the region in the x, y plane bounded by the graph of $y = \left(1 + (2 - x)^4\right)^{1/4}$, the x-axis, and the lines $x = 0$ and $x = 2$.



(a) Write an integral giving the value of the volume of the solid whose base is the given region and whose cross-sections perpendicular to the x-axis are squares. (A three-dimensional view of the solid is presented in the figure.)

$$\text{Volume} = \int_0^2 y^2\,dx = \int_0^2 \left(1 + (2 - x)^4\right)^{1/2} dx\,.$$

(b) Explain how Riemann sum approximations of your integral are related to approximations of the volume of the solid.

If we slice through the figure with a plane perpendicular to the x-axis that meets the axis at the point $(x, 0, 0)$, then we are given that this slice is a square with side $y = y(x) = \left(1 + (2 - x)^4\right)^{1/4}$. Therefore, it has area equal to $A(x) = y^2 = \left(1 + (2 - x)^4\right)^{1/2}$. If the solid is sliced into n thin slices by such planes spaced Δx apart, each with thickness equal to Δx, then the volume of the slice through x is approximately equal to $A(x)\Delta x = \left(1 + (2 - x)^4\right)^{1/2}\Delta x$. Adding up these numbers over all slices gives us $\sum_{i=1}^{n} \left(1 + (2 - x_i)^4\right)^{1/2}\Delta x$, which is a Riemann sum approximation to the integral of part (a). It is also a good approximation to the sum of the volumes of the slices, i.e. the volume of the solid. Taking the limit as the number n of slices tends to infinity then gives us that the integral is equal to the volume.

(c) Find, as accurately as you can, the value of the volume of the solid described in part (a). Explain how you computed your answer.

The integral was calculated using the numerical integration function on a TI-83 calculator with the function $y(x) = (1 + (2 - x)^4)^{1/2}$ as integrand and $x = 0$, $x = 2$ as the end points. The volume is approximately **3.65348**.