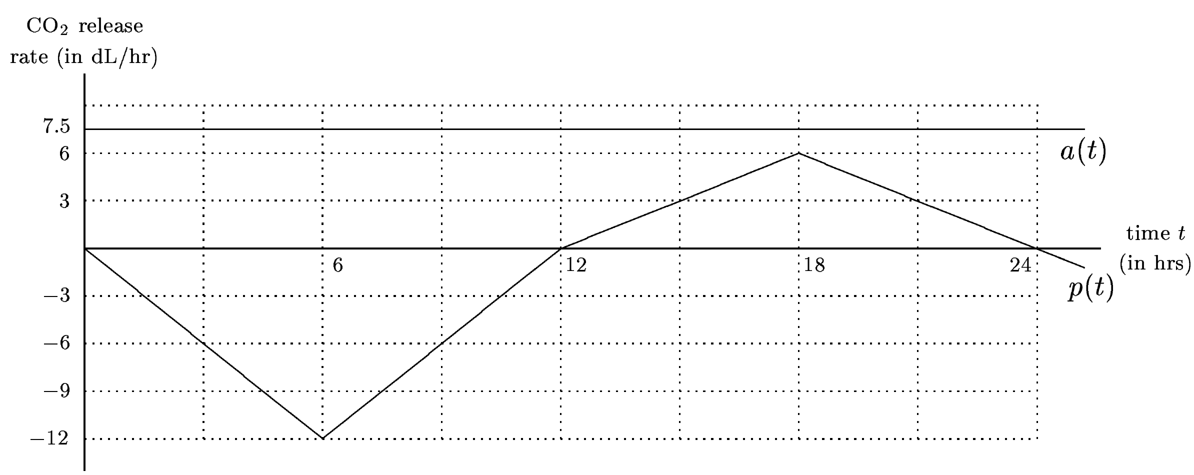
8. (14 points) A team of biologists seeking to develop alternate solutions to the use of pesticides proposes the following experiment. A plant infested by a large colony of aphids (small insects), is placed in a container originally saturated in dioxygen (O_2). They hope that the parasites can be suffocated by the CO_2 produced by their natural activities (e.g. breathing) and those of the plant (e.g. photosynthesis). Make the simplifying assumptions, which are approximations, that:

(i) The container is sealed; no molecules enter or leave.

(ii) The sum of the volumes of O_2 and CO_2 in the container is constant and equal to 117 deciliters (dL); i.e. 11.7 liters.

(iii) At the start of the experiment ($t = 0$), there is no CO_2 in the container.

The light in the room is adjusted so as to mimick a "perfect" 24-hour period. The parasites produce CO_2 at a constant rate of 7.5 dL per hour. When the light is turned on, the plant absorbs CO_2 and releases O_2. When the light is turned off, it does the opposite; i.e. uses O_2 and produces CO_2. The rates of release in the container of CO_2 (in dL per hour) for the parasites, $a(t)$, and the plant, $p(t)$, are plotted below. Note that negative rates correspond to absorption.



(a) By approximately how much does the volume $V(t)$ of CO_2 in the container change in the small time interval between t and $t + \Delta t$? Express your answer in terms of Δt, and of the rates $p(t)$ and $a(t)$.

The production of CO_2 in the tank comes from both the aphids and the plant. So the total rate at which CO_2 is released is given by $a(t) + p(t)$. Thus, in the small time interval from t to $t + \Delta t$, the (small) change in the volume of CO_2 is :

$$\mathbf{\Delta V = \big(a(t) + p(t)\big)\,\Delta t}\,.$$

Solution continued on next page.

Solution continued from previous page.

(b) Write an integral that gives the volume $V(12)$ of CO_2 in the container after 12 hours. Explain why your integral gives the value of $V(12)$. You will probably want to use the answer from **(a)** in your explanation.

We are told that there is no CO_2 in the tank at time $t = 0$, so $V(0) = 0$. To find the volume of CO_2 present in the tank after 12 hours, i.e. $V(12)$, we can "slice" the interval from $t = 0$ to $t = 12$ into small time intervals of length Δt. According to part **(a)**, *during each of these small time intervals, the change in the volume of carbon dioxyde is $\Delta V = \big(a(t) + p(t)\big)\Delta t$. Summing all those little contributions gives a Riemann sum. Letting Δt go to zero, the Riemman sum gives the integral:*

$$\mathbf{V(12)} = \int_0^{12} \big(\mathbf{a(t)} + \mathbf{p(t)}\big)\, \mathbf{dt}\,.$$

(c) What is the value of $V(12)$? Explain how you obtained your answer.

The integral in **(b)** *represents the sum of the area under the graph of $a(t)$ and the area under the graph of $p(t)$, between $t = 0$ and $t = 12$. To find its value, it suffices to count the "boxes" on the chart. The scale on the figure shows that each box represents a volume of 9 dL of CO_2. Counting the boxes gives:*

$$\mathbf{V(12)} = (\mathbf{10} - \mathbf{8}) \times \mathbf{9} = \mathbf{18}\ \textbf{\textit{deciliters.}}$$

(d) When the container becomes saturated in CO_2, i.e. no O_2 remains, the parasites suffocate to death, and the experiment is stopped. Decide whether that will happen within the 24-hour period; and, if you think it will, estimate this time.

We are told there are 117 dL of O_2 available at $t = 0$. All aphids will thus die if the the volume of CO_2 in the container reaches the value 117. To decide whether that will happen before the end of the experiment, i.e. before $t = 24$ hours, we evaluate $V(24)$ by counting the boxes as was done in **(c)**. *Doing so gives:*

$$V(24) = (20 - 8 + 4) \times 9 = 144 > 117 \ \textit{deciliters.}$$

Accordingly, the aphids **will** *indeed suffocate to death before the experiment ends.*
Again counting the boxes, we find that will happen after **21 hours**. *Indeed, at $t = 21$ the total volume of CO_2 in the container is:*

$$\mathbf{V(21)} = (\mathbf{17.5} - \mathbf{8} + \mathbf{3.5}) \times \mathbf{9} = \mathbf{117}\ \textbf{\textit{deciliters.}}$$