

7. (3 points each) Multiple choice. Circle *the single* correct answer to each one of the following questions. (No partial credit will be awarded.)

(I) Suppose that f and g are positive, continuous functions on the interval $x \geq 1$ such that $f(x) \geq g(x)$ for $1 \leq x < 4$ and $f(x) \leq g(x)$ for $x \geq 4$. If $\int_4^\infty g(x) dx$ converges to the value A , then what can you say about $\int_1^\infty f(x) dx$?

- (a) It may diverge or converge; it cannot be determined.
- (b) It converges to a number that must be greater than A .
- (c) It converges to a number that must be less than A .
- (d) It converges to a number that may be greater than A or may be less than A .

(II) Suppose that $\int_3^\infty \frac{100}{x^p} dx$ diverges for some $p \neq 1$. What can you conclude about $\int_0^3 \frac{100}{x^p} dx$?

- (a) It also diverges.
- (b) It converges.
- (c) It may diverge or converge; it cannot be determined.
- (d) It is not an improper integral, so we don't talk about its convergence or divergence.

(III) Which of the following is true about $\int_2^\infty \frac{1}{\sqrt{\theta} + \theta^2} d\theta$?

- (a) It converges, by comparison with $\int_2^\infty 1/\theta^2 d\theta$.
- (b) It diverges, by comparison with $\int_2^\infty 1/\sqrt{\theta} d\theta$.
- (c) It diverges, by comparison with $\int_2^\infty 1/\theta^2 d\theta$.
- (d) It converges, by comparison with $\int_2^\infty 1/\sqrt{\theta} d\theta$.

(IV) Suppose that $\int_1^\infty A(t) dt$ converges, and $A(t)$ is a positive, continuous function for all real numbers. Then, what can you say about the convergence or divergence of

$$\int_1^\infty (A(t) + A(t)) dt \quad \text{and} \quad \int_1^\infty \left(A(t) + \frac{1}{t} \right) dt?$$

- (a) $\int_1^\infty (A(t) + A(t)) dt$ converges, but $\int_1^\infty (A(t) + 1/t) dt$ diverges.
- (b) $\int_1^\infty (A(t) + A(t)) dt$ diverges, but $\int_1^\infty (A(t) + 1/t) dt$ converges.
- (c) They both diverge.
- (d) They both converge.