1. (8 points) Consider the functions f and g defined below. Assume a is a nonzero constant.

$$f(x) = \frac{(x-a)^2}{\sqrt{x}}\ , \qquad g(x) = x\cos(ax),$$

(a) (4 pts.) Find the family of antiderivatives of $f(x)$. Show step-by-step work.

We have

$$f(x) = \frac{x^2 - 2ax + a^2}{\sqrt{x}} = x^{3/2} - 2ax^{1/2} + a^2x^{-1/2}.$$

So,

$$\int f(x)\ dx = \frac{2}{5}x^{5/2} - \frac{4}{3}ax^{3/2} + 2a^2x^{1/2} + C.$$

(b) (4 pts.) Find the family of antiderivatives of $g(x)$. Show step-by-step work.

Using integration by parts,

$$\begin{aligned} u &= x & u' &= 1 \\ v' &= \cos(ax) & v &= \frac{\sin(ax)}{a}. \end{aligned}$$

Then,

$$\begin{aligned} \int x\cos(ax)\ dx =& \frac{x}{a}\sin(ax) - \int \frac{\sin(ax)}{a}\ dx \\ =& \frac{x}{a}\sin(ax) + \frac{\cos(ax)}{a^2} + C. \end{aligned}$$