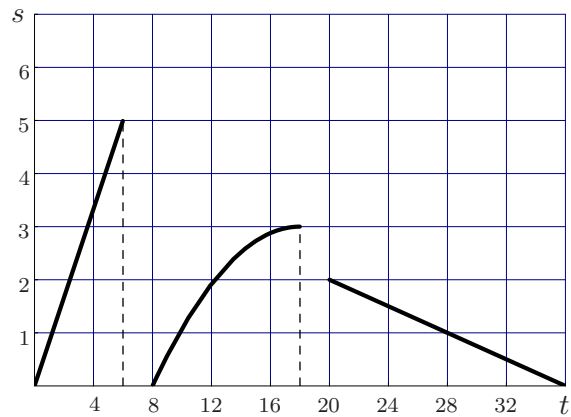


2. (13 points) In the magical tale of Harry Potter and the Half-blood Prince, Harry and professor Dumbledore go in search of a horcrux, a dark magic device created by the dark wizard Voldemort in order to hide and preserve a piece of his soul.

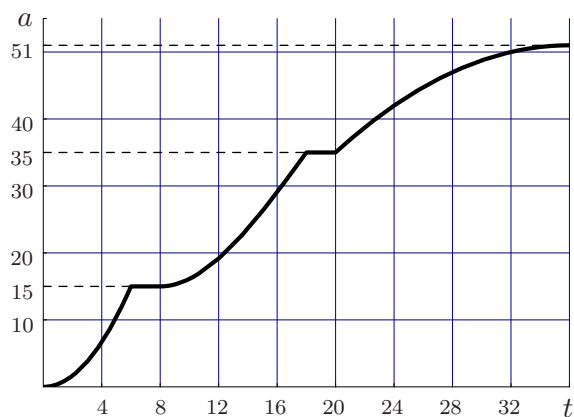
To get to the horcrux, Dumbledore must drink a lethal green elixir filled with dark power. Harry's task is to scoop some of the magic liquid into a goblet, feed it to Dumbledore, then turn around and scoop some more liquid into the goblet, repeating these steps until the elixir is gone.

Harry notices that, after the first glass, Dumbledore's drinking speed increases at a decreasing rate. By the end, Dumbledore is drinking as slowly as possible.

The graph below represents Dumbledore's drinking speed, s (in fluid ounces per second) against time, t (in seconds.)



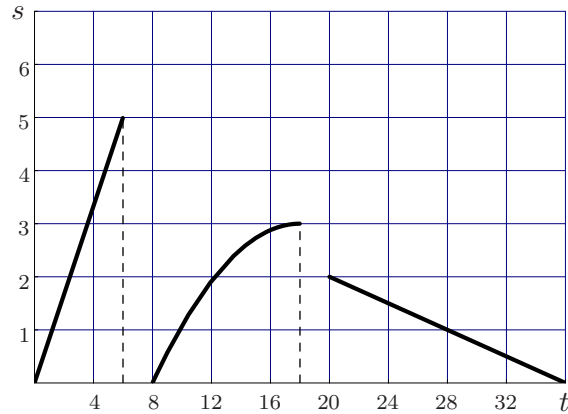
- (a) (5 pts.) Sketch the amount a of the elixir that Dumbledore has drunk as a function of time t during the first 32 seconds after he started drinking. Include scales on each axis, and label them appropriately. Clearly indicate on your sketch the heights of the graph of $a(t)$ at $t = 6$, $t = 18$ and $t = 36$.



(This problem continues on the next page.)

(This is a continuation of Problem 2. For your convenience, the original graph is reprinted here.)

The graph below represents Dumbledore's drinking speed, s (in fluid ounces per second) against time, t (in seconds).



- (b) (4 pts.) Write an expression that represents Dumbledore's average drinking speed during the first 12 seconds after he starts drinking. Use the graph to estimate this value. *Show step-by-step work.*

$$\begin{aligned} \frac{1}{12} \int_0^{12} s(t) dt &= \frac{1}{12} \left[\int_0^6 s(t) dt + \int_8^{12} s(t) dt \right] \\ &\simeq \frac{1}{12} [15 + 4] \\ &= \frac{19}{12} \text{ fluid oz./second} \end{aligned}$$

- (c) (4 pts.) *Exactly* how long does it take Dumbledore to drink the last 4 fluid ounces of the dark-magic substance? *Briefly explain your work.*

We are looking for a number b such that

$$\int_{36-b}^{36} s(t) dt = 4.$$

The last section of the graph is linear, so the area underneath the graph is represented by a triangle. We can see from the graph that if $b = 8$, then the area from $t = 28$ to $t = 36$ is $(1/2) 8 = 4$.

It follows that $b = 8$ (seconds) or, in words, it takes Dumbledore exactly 8 seconds to drink the last 4 oz. of liquid.