5. (11 points) A child is sitting on a Ferris wheel. If the origin is at the center of the circle and we measure x and y in meters, her motion is given by the following parametric equations:

$$x = 125\sin((2\pi/9)t), \qquad y = -125\cos((2\pi/9)t),$$

where we measure t in minutes since she boarded the ride.

(a) (2 pts.) What is the diameter of the Ferris wheel?

$$D = 250 \text{ meters}$$

(b) (2 pts.) How long does it take for the Ferris wheel to make one complete revolution?

9 minutes

(c) (3 pts.) Find the speed of the child 10 minutes into the ride.

$$\frac{dx}{dt} = \frac{2\pi}{9}125\ \cos\left(\frac{2\pi}{9}t\right), \qquad \frac{dy}{dt} = \frac{2\pi}{9}125\ \sin\left(\frac{2\pi}{9}t\right)$$

$$S = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(\frac{2\pi}{9}\ 125\right)^2} \simeq 87.26 \text{ meters/minute}$$

(d) (4 pts.) If at 10 minutes into the ride the child were to suddenly step off of the Ferris wheel, her motion would initially be along the tangent line at $t = 10$. Determine parametric equations for this tangent line.

We have,

$$x(10) = 125\sin\left(\frac{2\pi}{9}\ 10\right) \simeq 80$$
$$y(10) = -125\cos\left(\frac{2\pi}{9}\ 10\right) \simeq -95.$$

Also,

$$\left.\frac{dx}{dt}\right|_{t=10} = \frac{2\pi}{9}\ 125\cos\left(\frac{2\pi}{9}\ 10\right) \simeq 67$$
$$\left.\frac{dy}{dt}\right|_{t=10} = \frac{2\pi}{9}\ 125\sin\left(\frac{2\pi}{9}\ 10\right) \simeq 56.$$

So,

$$x(t) = 67\ t + 80$$
$$y(t) = 56\ t - 95.$$