7. (3 points each) Multiple choice. Circle *the single* correct answer to each one of the following questions. (No partial credit will be awarded.)

(I) Suppose that f and g are positive, continuous functions on the interval $x \geq 1$ such that $f(x) \geq g(x)$ for $1 \leq x < 4$ and $f(x) \leq g(x)$ for $x \geq 4$. If $\int_4^\infty g(x)\,dx$ converges to the value A, then what can you say about $\int_1^\infty f(x)\,dx$?

(a) It may diverge or converge; it cannot be determined.

(b) It converges to a number that must be greater than A.

(c) It converges to a number that must be less than A.

$\boxed{\text{(d)}}$ It converges to a number that may be greater than A or may be less than A.

(II) Suppose that $\int_3^\infty \frac{100}{x^p}\,dx$ diverges for some $p \neq 1$. What can you conclude about $\int_0^3 \frac{100}{x^p}\,dx$?

(a) It also diverges.

$\boxed{\text{(b)}}$ It converges.

(c) It may diverge or converge; it cannot be determined.

(d) It is not an improper integral, so we don't talk about its convergence or divergence.

(III) Which of the following is true about $\displaystyle\int_2^\infty \frac{1}{\sqrt{\theta}+\theta^2}\,d\theta$?

$\boxed{\text{(a)}}$ It converges, by comparison with $\int_2^\infty 1/\theta^2\,d\theta$.

(b) It diverges, by comparison with $\int_2^\infty 1/\sqrt{\theta}\,d\theta$.

(c) It diverges, by comparison with $\int_2^\infty 1/\theta^2\,d\theta$.

(d) It converges, by comparison with $\int_2^\infty 1/\sqrt{\theta}\,d\theta$.

(IV) Suppose that $\int_1^\infty A(t)\,dt$ converges, and $A(t)$ is a positive, continuous function for all real numbers. Then, what can you say about the convergence or divergence of

$$\int_1^\infty (A(t)+A(t))\,dt \quad \text{and} \quad \int_1^\infty \left(A(t)+\frac{1}{t}\right)\,dt?$$

$\boxed{\text{(a)}}$ $\int_1^\infty (A(t)+A(t))\,dt$ converges, but $\int_1^\infty (A(t)-+1/t)\,dt$ diverges.

(b) $\int_1^\infty (A(t)+A(t))\,dt$ diverges, but $\int_1^\infty (A(t)+1/t)\,dt$ converges.

(c) They both diverge.

(d) They both converge.