7. (3 points each) Multiple choice. Circle the single correct answer to each one of the following questions. (No partial credit will be awarded.)

(I) Suppose that $f$ and $g$ are positive, continuous functions on the interval $x \geq 1$ such that $f(x) \geq g(x)$ for $1 \leq x < 4$ and $f(x) \leq g(x)$ for $x \geq 4$. If $\int_a^\infty g(x) \, dx$ converges to the value $A$, then what can you say about $\int_1^\infty f(x) \, dx$?

(a) It may diverge or converge; it cannot be determined.
(b) It converges to a number that must be greater than $A$.
(c) It converges to a number that must be less than $A$.  
(d) It converges to a number that may be greater than $A$ or may be less than $A$.

(II) Suppose that $\int_3^\infty \frac{100}{x^p} \, dx$ diverges for some $p \neq 1$. What can you conclude about $\int_0^\infty \frac{1}{x^p} \, dx$?

(a) It also diverges.
(b) It converges.
(c) It may diverge or converge; it cannot be determined.
(d) It is not an improper integral, so we don’t talk about its convergence or divergence.

(III) Which of the following is true about $\int_2^\infty \frac{1}{\sqrt{\theta} + \theta^2} \, d\theta$?

(a) It converges, by comparison with $\int_2^\infty \frac{1}{\theta^2} \, d\theta$.
(b) It diverges, by comparison with $\int_2^\infty \frac{1}{\sqrt{\theta}} \, d\theta$.
(c) It diverges, by comparison with $\int_2^\infty \frac{1}{\theta^2} \, d\theta$.
(d) It converges, by comparison with $\int_2^\infty \frac{1}{\sqrt{\theta}} \, d\theta$.

(IV) Suppose that $\int_1^\infty A(t) \, dt$ converges, and $A(t)$ is a positive, continuous function for all real numbers. Then, what can you say about the convergence or divergence of

$$\int_1^\infty (A(t) + A(t)) \, dt \quad \text{and} \quad \int_1^\infty \left(A(t) + \frac{1}{t}\right) \, dt?$$

(a) $\int_1^\infty (A(t) + A(t)) \, dt$ converges, but $\int_1^\infty (A(t) + 1/t) \, dt$ diverges.
(b) $\int_1^\infty (A(t) + A(t)) \, dt$ diverges, but $\int_1^\infty (A(t) + 1/t) \, dt$ converges.
(c) They both diverge.
(d) They both converge.