

8. (10 points) Circle City is a circular city with a radius of five miles. A straight highway runs East-West through the center of the city. The density of the population at a distance y (in miles North or South) from the road is well approximated by

$$D(y) = 4y$$

(in thousands of people per square mile.) Apparently many people in Circle City like to live as far from the highway as possible.

- (a) (4 pts.) Write a Riemann sum that approximates the total population of Circle City.

Since the density depends only on y , it is nearly constant on horizontal slices. Slicing horizontally, we have,

$$\text{Area of a slice} \simeq 2r\Delta y, \quad \text{where } r = \sqrt{25 - y^2},$$

and so,

$$\text{Total Population of a slice} \simeq 8y\sqrt{25 - y^2} \Delta y.$$

Summing up, we have

$$2 \sum 8y\sqrt{25 - y^2} \Delta y,$$

where the factor “2” is due to the symmetry of areas and density with respect to the highway.

- (b) (4 pts.) Write an integral that gives the total population of Circle City.

$$2 \int_0^5 8y\sqrt{25 - y^2} dy$$

- (c) (2 pts.) Evaluate your integral to find the total population of Circle City.

Using integration by substitution, or a calculator, we see that the total population of Circle City is

$$2 \int_0^5 8y\sqrt{25 - y^2} dy = \frac{16}{3}(25)^{3/2} \simeq 667 \text{ thousand people.}$$