

9. [10 points] To improve their understanding of the material in their Calculus course, Alex and Chris have invented a set of statements about the material they have been studying. These statements are given below. For each statement, circle **true** (that is, the statement is always true), or **false** (it isn't), and give a one sentence explanation for your answer.

(a) [2 of 10 points] If a bounded continuous function $f(x)$ has the properties that $f(x) > \frac{1}{x}$ for $1 < x < 50$, $f(50) = \frac{1}{50}$, and $f(x) < \frac{1}{x}$ for $x > 50$, then $\int_1^\infty f(x) dx$ converges.

TRUE FALSE

(b) [2 of 10 points] If a bounded continuous function $f(x)$ has the properties that $f(x) > \frac{1}{x^2}$ for $1 < x < 50$, $f(50) = \frac{1}{2500}$, and $f(x) < \frac{1}{x^2}$ for $x > 50$, then $\int_1^\infty f(x) dx$ converges.

TRUE FALSE

(c) [2 of 10 points] Since the function $\frac{\sin(x)+2}{\sqrt{x}}$ is always less than $\frac{3}{\sqrt{x}}$ for $2 \leq x < \infty$ and $\lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} = 0$, we know that $\int_2^\infty \frac{\sin(x)+2}{\sqrt{x}} dx$ converges.

TRUE FALSE

(d) [2 of 10 points] If $0 < \frac{1}{x} < g(x) < \frac{1}{x^2}$ for $0 < x < 1$, then the area between $g(x)$ and the x -axis for $0 < x < 1$ is guaranteed to be finite.

TRUE FALSE

(e) [2 of 10 points] Let $f(x) = \frac{1}{(x-1)^2}$. Then if $F(x) = \int_0^x f(t) dt$, we know that $F(0) = 0$ and that $F(2) = \int_0^2 \frac{1}{(t-1)^2} dt = -\frac{1}{t-1} \Big|_0^2 = -1 - 1 = -2$.

TRUE FALSE