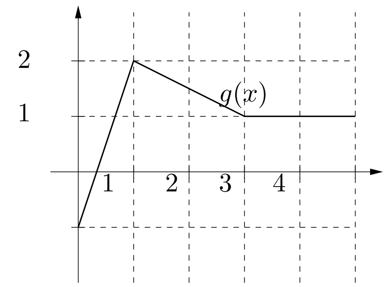
1. [12 points] For all of parts (a)–(d), let $f(x) = 2x - 4$ and let $g(x)$ be given in the graph to the right.



(a) [3 points of 12] Find $\int_1^5 g'(x)\,dx$.

Solution:
By the Fundamental Theorem of Calculus,

$$\int_0^5 g'(x)\,dx = g(5) - g(1) = 1 - (2) = -1.$$

Alternately, note that $g'(x) = 3$ for $0 < x < 1$, $g'(x) = -\frac{1}{2}$ for $1 < x < 3$, and $g'(x) = 0$ for $x > 3$. Thus $\int_1^5 g'(x)\,dx = \int_1^3 -\frac{1}{2}\,dx = -1$.

(b) [3 points of 12] Find $\int_0^5 g(x)\,dx$.

Solution:
This integral is just the area between the graph of $g(x)$ and the x-axis between $x = 0$ and $x = 5$, counting area below the axis as negative. This is

$$\int_0^5 g(x)\,dx = -\,(\text{area between 0 and } 2/3) + (\text{area between 0 and 1}) + (\text{area between 1 and 3}) + (\text{area between 3 and 5}) = -\left(\frac{1}{2}(\frac{1}{3})(1)\right) + \left(\frac{1}{2}(\frac{2}{3})(2)\right) + (3) + (2) = 5\frac{1}{2}.$$

(c) [3 points of 12] Find $\int_2^{4.5} g(f(x))\,dx$.

Solution:
Using substitution with $w = f(x) = 2x - 4$, we have $\frac{1}{2}dw = dx$, so that $\int_2^{4.5} g(f(x))\,dx = \frac{1}{2}\int_{w(2)}^{w(4.5)} g(w)\,dw = \frac{1}{2}\int_0^5 g(w)\,dw$. But the calculation above gives $\int_0^5 g(w)\,dw = 5\frac{1}{2} = \frac{11}{2}$, so $\int_2^{4.5} g(f(x))\,dx = \frac{1}{2}\cdot\frac{11}{2} = \frac{11}{4} = 2.75$.

(d) [3 points of 12] Find $\int_0^5 f(x)\cdot g'(x)\,dx$.

Solution:
Using integration by parts with $u = 2x - 4$ and $v' = g'$, we have $u' = 2$ and $v = g$, so that

$$\begin{aligned}\int_0^5 f(x)\cdot g'(x)\,dx &= (2x-4)\cdot g(x)\Big|_0^5 - 2\int_0^5 g(x)\,dx \\ &= (6)(1) - (-4)(-1) - 2(\frac{11}{2}) = 2 - 11 = -9.\end{aligned}$$

Alternate solution: note that $g'(x) = 3$ for $0 < x < 1$, $g'(x) = -\frac{1}{2}$ for $1 < x < 3$, and $g'(x) = 0$ for $x > 3$. Thus

$$\begin{aligned}\int_0^5 f(x)\cdot g'(x)\,dx &= \int_0^1 3(2x-4)\,dx + \int_1^3 -\frac{1}{2}(2x-4)\,dx \\ &= 3(x^2 - 4x)\big|_0^1 + (-\frac{x^2}{2} + 2x)\big|_1^3 = -9 + (\frac{3}{2} - \frac{3}{2}) = -9.\end{aligned}$$