2. [12 points] While working on their team homework, Alex and Chris find that they have evaluated the same integral—but that they each used a different method, and got different answers! Alex found

$$\int (2x-1)(3+x)^4\,dx = (2x-1)\left(\frac{1}{5}(3+x)^5\right) - \frac{1}{15}(3+x)^6 + C.$$

while Chris had

$$\int (2x-1)(3+x)^4\,dx = \frac{1}{3}(3+x)^6 - \frac{7}{5}(3+x)^5 + C,$$

(a) [6 of 12 points] Considering the form of the solution that Alex found, what method is it most likely that Alex used? Use this method and verify that you obtain the same solution.

Solution:
We notice that the first term of Chris' solution, $(2x-1)(\frac{1}{5}(3+x)^5)$, is the product uv if $u = 2x-1$ and $v' = (3+x)^4$, so it looks as if this solution was obtained by using integration by parts. With these choices of u and v', we have $u' = 2$ and $v = \frac{1}{5}(3+x)^5$, so

$$\begin{aligned}\int (2x-1)(3+x)^4\,dx &= (2x-1)\left(\frac{1}{5}(3+x)^5\right) - \int \frac{2}{5}(3+x)^5\,dx\\ &= (2x-1)\left(\frac{1}{5}(3+x)^5\right) - \frac{1}{15}(3+x)^6 + C.\end{aligned}$$

(b) [6 of 12 points] Considering the form of the solution that Chris found, what method is it most likely that Chris used? Use this method and verify that you obtain the same solution.

Solution:
We see only factors of $3+x$ to various powers in the solution, which suggests that Alex may have used substitution with $w = 3+x$. This works because then $dw = dx$ and $2x-1 = 2w-7$, so that

$$\begin{aligned}\int (2x-1)(3+x)^4\,dx &= \int (2w-7)w^4\,dw = \int 2w^5 - 7w^4\,dw\\ &= \frac{2}{6}w^6 - \frac{7}{5}w^5 + C = \frac{1}{3}(3+x)^6 - \frac{7}{5}(3+x)^5 + C.\end{aligned}$$