2. [12 points] While working on their team homework, Alex and Chris find that they have evaluated the same integral—but that they each used a different method, and got different answers! Alex found
\[ \int (2x - 1)(3 + x)^4 \, dx = (2x - 1) \left( \frac{1}{3}(3 + x)^5 \right) - \frac{1}{15}(3 + x)^6 + C. \]
while Chris had
\[ \int (2x - 1)(3 + x)^4 \, dx = \frac{1}{3}(3 + x)^6 - \frac{7}{5}(3 + x)^5 + C, \]

(a) [6 of 12 points] Considering the form of the solution that Alex found, what method is it most likely that Alex used? Use this method and verify that you obtain the same solution.

Solution:
We notice that the first term of Chris’ solution, \((2x - 1)(\frac{1}{3}(3 + x)^5)\), is the product \(uv\) if \(u = 2x - 1\) and \(v' = (3 + x)^4\), so it looks as if this solution was obtained by using integration by parts. With these choices of \(u\) and \(v'\), we have \(u' = 2\) and \(v = \frac{1}{3}(3 + x)^5\), so
\[ \int (2x - 1)(3 + x)^4 \, dx = (2x - 1) \left( \frac{1}{3}(3 + x)^5 \right) - \int \frac{2}{3}(3 + x)^5 \, dx \]
\[ = (2x - 1) \left( \frac{1}{3}(3 + x)^5 \right) - \frac{1}{15}(3 + x)^6 + C. \]

(b) [6 of 12 points] Considering the form of the solution that Chris found, what method is it most likely that Chris used? Use this method and verify that you obtain the same solution.

Solution:
We see only factors of \(3 + x\) to various powers in the solution, which suggests that Alex may have used substitution with \(w = 3 + x\). This works because then \(dw = dx\) and \(2x - 1 = 2w - 7\), so that
\[ \int (2x - 1)(3 + x)^4 \, dx = \int (2w - 7)w^4 \, dw = \int 2w^5 - 7w^4 \, dw \]
\[ = \frac{2}{6}w^6 - \frac{7}{5}w^5 + C = \frac{1}{3}(3 + x)^6 - \frac{7}{5}(3 + x)^5 + C. \]