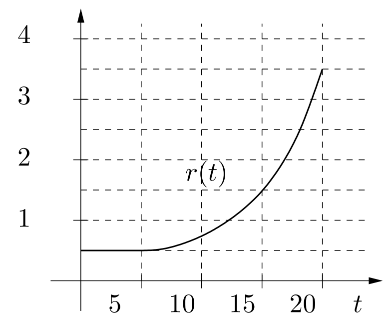
3. [12 points] Having completed their team homework, Alex and Chris are making chocolate chip cookies to celebrate. The rate at which they make their cookies, $r(t)$, is given in cookies/minute in the figure to the right (in which t is given in minutes). After $t =$ 20 minutes they have completed their cookie making extravaganza.



(a) [3 of 12 points] Write an expression for the total number of cookies that they make in the 20 minutes they are baking. Why does your expression give the total number of cookies?

Solution:
We are given the rate at which the cookies are being produced, so we know by the Fundamental Theorem of Calculus that the total number of cookies produced is given by $\int_0^{20} r(t)\,dt$.

(b) [3 of 12 points] Using $\Delta t = 5$, find left- and right-Riemann sum and trapezoid estimates for the total number of cookies that they make.

Solution:
Using $\Delta t = 5$, the left- and right-hand Riemann sums are

$$\text{LEFT}(4) \approx 5(0.5 + 0.5 + 0.75 + 1.5) = 16.25$$
$$\text{RIGHT}(4) \approx 5(0.5 + 0.75 + 1.5 + 3.5) = 31.25.$$

Thus the trapezoid estimate is $\text{TRAP}(4) = \frac{1}{2}(16.25 + 31.25) = 23.75$, or about 24 cookies.

(c) [3 of 12 points] How large could the error in each of your estimates in (b) be?

Solution:
We know that the maximum error in the left- or right-hand sums is just $\Delta t(r(20) - r(0)) = 5(3.5 - 0.5) = 15$ cookies. The maximum error in the trapezoid estimate is half this, or 7.5 cookies.

(d) [3 of 12 points] How would you have to change the way you found each of your estimates to reduce the possible errors noted in (c) to one quarter of their current values?

Solution:
The error in the left- or right-hand sums drops as n, so we would have to take four times as many steps, reducing Δt to $\frac{5}{4} = 1.25$ min to reduce the error in those estimates by a factor of four. The error in the trapezoid estimate drops as n^2, the square of the number of steps we take in the calculation, so if we recalculated our estimate with $\Delta t = 2.5$ minutes it would be four times as accurate.