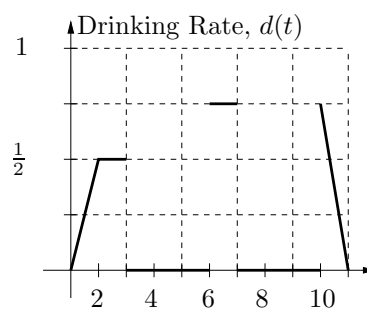
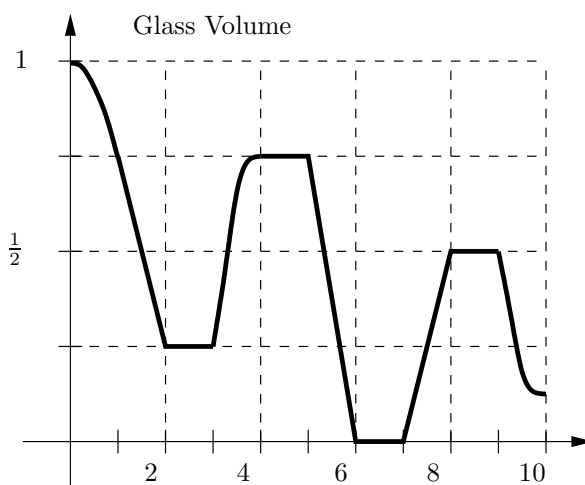
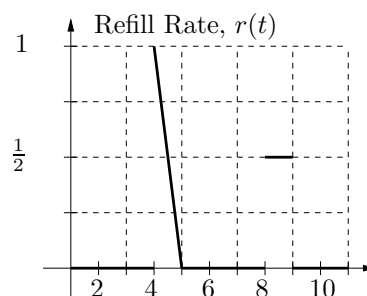


7. [16 points] As they eat their cookies, Alex and Chris are drinking milk. They each have a glass from which they are drinking, and the jug of milk is conveniently located in the middle of the table. They both start with a full glass of milk (one pint). As they are drinking, Chris sneakily refills the glass that Alex is drinking from while Alex is distracted by the student that is passing by. The rate at which Alex is drinking the milk, $d(t)$, is shown in the top figure to the right, while the rate at which Chris refills the glass, $r(t)$, is shown in the bottom figure. Both figures give the rates in pints per minute, and the time in minutes.



- (a) [8 of 16 points] Let $V(t)$ be the amount (volume) of milk in the glass that Alex is drinking from, as a function of time. Carefully sketch $V(t)$ on the axes provided. Be sure to label your axes and that your sketch has accurate vertical and horizontal scales.



Solution:

To find the amount of milk in the glass, start with the initial volume ($V = 1$ pint), and use the Fundamental Theorem of Calculus to calculate the change in volume given the rates. The area under the top rate curve (the rate at which Alex is drinking) results in a decrease in the volume of the glass, while the area under the bottom (the rate at which Chris is refilling the glass) increases the volume. This results in the bottom graph shown above

- (b) [2 of 16 points] Does Alex ever finish all of the milk in the glass?

Solution:

Alex empties the glass at time $t = 6$ minutes, but Chris refills it to half full starting at $t = 7$.