8. [12 points] In class, Chris’ calculus professor is well known to cover material at a rate \( m(t) = \frac{1}{12(t-20)^{2/3}} \) textbook sections/minute, where \( t \) is the time in minutes since the start of class.

(a) [2 of 12 points] What is the meaning of the integral \( \int_0^{80} m(t) \, dt \) (include units in your explanation)?

Solution:
The integral is the area under the rate \( m(t) \) between \( t = 0 \) and \( t = 80 \), which, by the Fundamental Theorem of Calculus is the total number of sections that Chris’ professor covers in the 80 minute (90 less ten) class period.

(b) [4 of 12 points] How many sections would you estimate the professor covers in the first minute of class? In the 20th minute? Why?

Solution:
We note that \( m(0) = \frac{1}{12(20)^{2/3}} = 0.0113 \) sections/minute. Thus we might guess that the professor might cover approximately 0.0113 sections-worth of material in the first minute. If we repeat this calculation for the 20th minute, we might guess that the number of sections covered is \((1 \text{ minute})(m(19)) = \frac{1}{12(1)^{2/3}} = \frac{1}{12} \) sections. Note, however, that \( m(20) \) is undefined—therefore, it appears that the professor is speaking at an infinite rate at about \( t = 20 \), so that we might also wonder if an infinite number of words are spoken. We can verify this by completing part (c) of this problem.

(c) [6 of 12 points] Find exactly (that is, by hand) the value of \( \int_0^{80} m(t) \, dt \).

Solution:
We note that because \( m(t) \) is discontinuous at \( t = 20 \) this is an improper integral. We therefore evaluate

\[
\int_0^{80} \frac{1}{12(t-20)^{2/3}} \, dt = \lim_{a \to 20^-} \int_0^a \frac{1}{12(t-20)^{2/3}} \, dt + \lim_{a \to 20^+} \int_a^{80} \frac{1}{12(t-20)^{2/3}} \, dt
\]

\[
= \lim_{a \to 20^-} \frac{1}{4} \left( \frac{1}{(t-20)^{1/3}} \right) \bigg|_0^a + \lim_{a \to 20^+} \frac{1}{4} \left( \frac{1}{(t-20)^{1/3}} \right) \bigg|_a^{80}
\]

\[
= \lim_{a \to 20^-} \frac{1}{4} \left( (a-20)^{1/3} + (20)^{1/3} \right) + \lim_{a \to 20^+} \frac{1}{4} \left( (60)^{1/3} - (a-20)^{1/3} \right)
\]

\[
= \frac{1}{4} (20)^{1/3} + \frac{1}{4} (60)^{1/3} \text{ sections.}
\]

Or, approximately 1.66 sections per class period. Because this integral is finite, it is clear that the amount of material covered in the 20th minute is, in fact, finite too.