

8. [12 points] The velocity of an object, with air resistance, may in some circumstances be given as

$$v(t) = \sqrt{\frac{g}{k}} \left( \frac{e^{2mt}}{e^{2mt} + 1} - \frac{1}{e^{2mt} + 1} \right),$$

where  $g$  is the acceleration due to gravity,  $k$  is a constant representing air resistance, and  $m = \sqrt{gk}$ .

- (a) [2 points of 12] Write an expression for the distance  $D$  that the object falls in the first  $t_0$  seconds.

- (b) [5 points of 12] Find the distance  $D$  (note that half of this calculation is significantly harder than the rest; do not waste too much time on it if you get stuck).

- (c) [5 points of 12] Suppose  $\sqrt{g/k} = 10$  and  $m = 1$ . Note that in this case  $v(3) = 9.95 \approx 10$ . Use a geometric argument to show that the distance traveled between  $t = 0$  and  $t = 3$ ,  $D(3)$ , satisfies the inequality  $15 < D(3) < 30$ .