1. [16 points] For this problem, $\int_1^5 g(x)\,dx = 12$, and $f(x) = 2x - 9$. Values of $g(x)$ are given in the table below.

x	1	2	3	4	5
$g(x)$	0.1	1.5	2	5	10

(a) [5 points of 16] Find $\int_5^7 g(f(x))\,dx$

Solution:
We use substitution with $w = f(x) = 2x - 9$. Then $w'(x) = 2$, so $dx = \frac{1}{2}dw$, and, noting that $w(5) = 1$ and $w(7) = 5$, $\int_5^7 g(f(x))\,dx = \int_1^5 \frac{1}{2} \cdot g(w)\,dw = \frac{1}{2}(12) = 6$.

(b) [5 points of 16] Find $\int_1^5 f(x) \cdot g'(x)\,dx$.

Solution:
We integrate by parts using $u = f(x) = 2x - 9$ and $v' = g'$. Then $u' = 2$ and $v = g$, so that

$$\int_1^5 f(x) \cdot g'(x)\,dx = f(x) \cdot g(x)\Big|_1^5 - \int_1^5 2g(x)\,dx = (1)g(5) + 7g(1) - 2(12)$$
$$= (10) + 7(0.1) - 24 = -13.3.$$

(c) [6 points of 16] Find $\int_1^5 \frac{g'(x)}{g(x)(g(x)+1)}\,dx$.

Solution:
First, substitute $w = g(x)$. Then $w' = g'(x)$, $w(1) = g(1) = 0.1$ and $w(5) = g(5) = 10$, so we get $\int_1^5 \frac{g'(x)}{g(x)(g(x)+1)}\,dx = \int_{0.1}^{10} \frac{1}{w(w+1)}\,dw$. We can find this by using the partial table of integrals or with partial fractions. From the table (the 8th equation) with $a = 0$ and $b = -1$, $\int_{0.1}^{10} \frac{1}{w(w+1)}\,dw = (\ln|w| - \ln|w+1|)\Big|_{0.1}^{10} = \ln(10) - \ln(0.1) - \ln(11) + \ln(1.1) = \ln(10) \approx 2.3$.
With partial fractions, $\frac{1}{w(w+1)} = \frac{A}{w} + \frac{B}{w+1}$ requires that $(A+B)w = 0$ and $A = 1$, so $B = -1$. This gives the result above.