4. [10 points] The Belgian scholar Lambert Quetelet published a distribution of chest measurements of Scottish soldiers in 1846. His distribution showed that the expected probability of a soldier having a chest measurement between 38 and 40 inches was given approximately by

$$P = \frac{1}{\sqrt{2\pi}} \int_{-0.9}^{0.1} e^{-y^2/2}\, dy.$$

Suppose that a scholar studying 19th century Scottish soldiers' physical measurements estimates this probability using the following calculation:

$$P = \frac{1}{\sqrt{2\pi}} \int_{-0.9}^{0.1} e^{-y^2/2}\, dy \approx \frac{1}{\sqrt{2\pi}}(0.25)(0.7406 + 0.8713 + 0.9629 + 0.9997) = 0.3565.$$

(a) [6 points of 10] What numerical method did the scholar use, how many steps were used in the method, and what was the step size?

Solution:
We know that there are four steps, because there are four terms in the sum given in the calculation. The step size is therefore $h = 0.25$, which we can also see from the factor of this h in the approximation. Then by trial and error we find that $e^{-(-0.775)^2/2} = e^{-(-0.9+0.125)^2/2} = 0.7406$, $e^{-(-0.525)^2/2} = e^{-(-0.9+0.375)^2/2} = 0.8713$, and so forth, so that the scholar has used the midpoint of each of the intervals $-9 \le x \le -0.65$, $-0.65 \le x \le -0.4$, etc., to calculate a "height" of the area approximating each interval. Thus the midpoint rule must have been used.

(b) [4 points of 10] Sketch a graph of $e^{-y^2/2}$. Explain how your graph indicates whether the scholar's approximation is an over- or under-estimate for the actual value of the integral.

Solution:
A graph of $e^{y^2/2}$ is shown to the right. This is concave down on the domain $-0.9 < y < 0.1$, so the midpoint rule, which can be visualized as summing boxes capped by the tangent to the curve, will be an overestimate for the actual value.