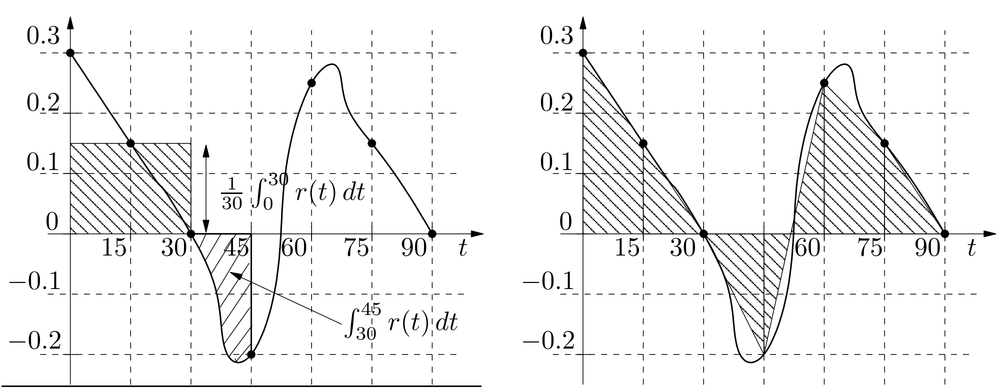
5. [12 points] Suppose that a student's rate of exam completion, given in problems per minute, is given by

t	0	15	30	45	60	75	90
$r(t)$	0.30	0.15	0	−0.20	0.25	0.15	0

(a) [6 points of 12] Write an integral that gives the total number of problems that the student completes in the 90 minute exam period. Estimate the number of problems that the student completes. Based on your estimate, does the student complete the exam?

Solution:
Because this is a rate of completion, the total number of problems completed is $N = \int_0^{90} r(t)\,dt$. We can estimate this with a left- or right-hand sum, and a better estimate is probably the average of these. A left-hand sum gives $N \approx (15)(0.30 + 0.15 + 0 - 0.20 + 0.25 + 0.15) = 9.75$, and a right-hand sum $N \approx (15)(0.15 + 0 - 0.2 + 0.25 + 0.15 + 0) = 5.25$. Averaging these, we estimate that the total number of problems completed is $\frac{1}{2}(9.75 + 5.25) = 7.5$. The problem was supposed to say that there were eight problems on the exam, but that somehow got deleted at some point. So it's *really* hard to tell if the student finished the exam. If there are eight problems on the exam, then the student might or might not have done so.

(b) [6 points of 12] The points on $r(t)$ given in (a) are shown in the two graphs below, connected by a smooth curve. On the first graph, illustrate the geometric meaning of (i) $\int_{30}^{45} r(t)\,dt$ and (ii) $\frac{1}{30}\int_0^{30} r(t)\,dt$. On the second graph, illustrate the geometric meaning of your calculation in (a).



Solution:
The solutions are graphed above. The integral $\int_{30}^{45} r(t)\,dt$ is the area between the curve and the x-axis; $\frac{1}{30}\int_0^{30} r(t)\,dt$ is the average height of the rate function for $0 \le t \le 30$, which is 0.15 (more accurately, it is the height for which the shaded box has the same area as the area under the curve). Our estimate from (a) is the trapezoid rule, which is shown on the graph to the right.