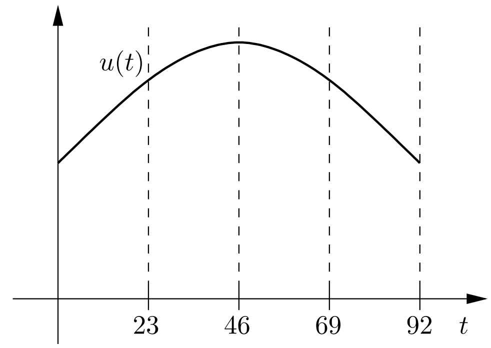
7. [14 points] Suppose that the rate of electricity use, $u(t)$, by a large calculus-problem producing factory is given, in megawatt-hours per day for the three months of summer, by the graph to the right. Time t is given in days from the beginning of the summer. As shown, the function $u(t)$ is symmetric about the middle of summer, July 16, which falls 46 days from the beginning of the summer. Let $M(T)$ be the *average rate of electricity use* of the factory over the course of the first T days of the summer.



(a) [6 points of 14] Write a formula for $M(T)$ in terms of $u(t)$.

Solution:
We know that the average value of a function $f(x)$ over an interval $a \le x \le b$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$, so the average energy consumption over the first T days is $M(T) = \frac{1}{T}\int_0^T u(t)\,dt$.

(b) [8 points of 14] Fill in the missing blanks of the following table of values of $M(T)$. Show your work, so that it is clear how you obtained your answers.

T	23	46	69	92
$M(T)$	8			11

Solution:
The average electricity use for $0 \le t \le 23$ is 8, so, by symmetry, the average use for $69 \le t \le 92$ must also be 8. Let the average use for $23 \le t \le 46$ (or, again because of symmetry, $46 \le t \le 69$) be x. Then, if the average for the entire range $0 \le t \le 92$ is to be 11, we must have $(2x + 16)/4 = 11$, so $x = 14$. Then the values $M(46) = \frac{1}{2}(8 + 14) = 11$ and $M(69) = \frac{1}{3}(8 + 28) = 12$.

Alternately, we know that $M(23) = 8$, so $\frac{1}{23}\int_0^{23} u(t)\,dt = 8$. Thus $\int_0^{23} u(t)\,dt = 184 = \int_{69}^{92} u(t)\,dt$. Then $M(92) = 11 = \frac{1}{92}\int_0^{92} u(t)\,dt$, so $\int_0^{92} u(t)\,dt = 1012$. But $\int_0^{92} u(t)\,dt = \int_0^{23} u(t)\,dt + \int_{23}^{69} u(t)\,dt + \int_{69}^{92} u(t)\,dt = 184 + \int_{23}^{69} u(t)\,dt + 184$. Thus $\int_{23}^{69} u(t)\,dt = 1012 - 368 = 644$, so $\int_{23}^{46} u(t)\,dt = \int_{46}^{69} u(t)\,dt = 322$. Thus $M(46) = \frac{1}{46}\int_0^{46} u(t)\,dt = \frac{1}{46}(184 + 322) = 11$, and $M(69) = \frac{1}{69}\int_0^{69} u(t)\,dt = \frac{1}{69}(\int_0^{23} u(t)\,dt + \int_{23}^{69} u(t)\,dt) = \frac{1}{69}(184 + 644) = 12$.

It is also worth noting that we can deduce $M(46) = 11$ from the symmetry of the graph: the average for the two halves of the data must be equal.