**4.** [16 points]   A University of Michigan squirrel, in Peru for a study-abroad semester, discovers a singularly symmetric pond hidden high in the Andes mountains. The pond has perfectly circular horizontal cross-sections, and its radii r at different depths y are shown (in meters) in the figure to the right, below. As shown, the outer edge of the pond has a radius of 5 meters, and the pond gets deeper towards its center.

a. [5 points] Set up an integral that gives the total volume of the pond. Your integral may involve the radius (r) and/or depth (y) of the pond. Be sure it is clear how you obtain your answer.

$y = 0,\ r = 5$

$y = 2.5,\ r = 0$

$y = 2,\ r = 0.5$

$y = 1.5,\ r = 1.5$

$y = 1,\ r = 2$

$y = 0.5,\ r = 3$
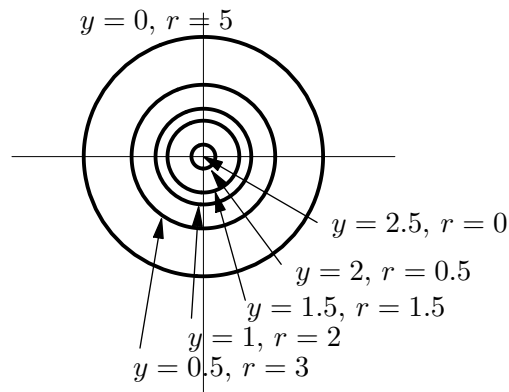
b. [5 points] Estimate the volume of the pond based on your work in (a).

c. [6 points] The pond is fed by a stream that is drying up as time goes on. If the stream delivers water to the pond at a rate of $r(t) = 60t\,e^{-t^2}$ m^3/year, does the pond ever fill? (Assume that the pond starts out empty when $t = 0$, and ignore other effects such as evaporation and rainfall.)