1. [10 points] A model¹ for the extinction rate of marine species during the Phanerozoic period (which extends from approximately 545 million years before the present until now) stipulates that this extinction rate, r(t), in numbers of marine animal families per million years, is

$$r(t) = \frac{3130}{t + 262},$$

where t is the number of million years after the start of the Phanerozoic period.

a. [5 points] Find an expression for E(t), the number of extinctions that occurred between the start of the Phanerozoic period and t million years thereafter.

Solution: Using the fundamental theorem of calculus, this is just

$$E(t) = \int_0^t \frac{3130}{x + 262} \, dx = (3130 \ln |x + 262|) \Big|_0^t = 3130 \ln(t + 262) - 3130 \ln(262).$$

(units are marine animal families).

b. [5 points] Find an expression for the average rate of extinctions between the start of the Phanerozoic period and t million years thereafter.

Solution: The average rate is

$$A(t) = \frac{1}{t-0} \int_0^t \frac{3130}{x+262} \, dx = \frac{1}{t} \left(3130 \ln(t+262) - 3130 \ln(262) \right)$$

(units are marine animal families/million years).

¹Newman & Eble, Decline in Extinction Rates and Scale Invariance in the Fossil Record, Paleobiology **25**:234–39 (1999)