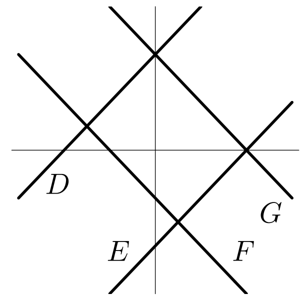
5. [6 points] Consider a parametric curve given by $x(t) = f(t)$, $y(t) = g(t)$, where $f(5) = 0$, $g(5) = 3$, $f'(5) > 0$ and $g'(5) < 0$. Which of the lines D, E, F, or G in the figure below could be the line tangent to the curve $(x(t), y(t))$ at $t = 5$? Explain.

Solution: Because $f'(5) > 0$ and $g'(5) < 0$, we know that the slope of the tangent line must be $\frac{dy}{dx} = \frac{g'(5)}{f'(5)} < 0$, so the tangent line must be one of F or G. Then we know that when $x(5) = f(5) = 0$, $y(5) = g(5) > 0$, so the y-intercept of the line must be positive, and we therefore know that the tangent must be line G.



6. [6 points] Find a set of inequalities in polar coordinates that describe the shaded triangle in the figure shown to the right, below.

Solution: The top boundary is $y = 1/2$, so on that boundary $r\sin(\theta) = 1/2$, or $r = \frac{1}{2\sin(\theta)}$. This holds for θ between $\theta = \arctan(\frac{1/2}{\sqrt{3}/2}) = \frac{\pi}{6}$ and $\theta = \frac{\pi}{2}$, so the inequalities are $\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$, $0 \le r \le \frac{1}{2\sin(\theta)}$. We could also say $\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$, $r \ge 0$, $r\sin\theta \le \frac{1}{2}$.

It is also correct to find equations for each of the boundaries and to then convert those to polar coordinates. This gives $r\sin\theta \le \frac{1}{2}$, $r\cos\theta \ge 0$ and $r\sin\theta \ge \frac{1}{\sqrt{3}} r\cos\theta$. These are, of course, equivalent to the preceding; the first is $r \le \frac{1}{2\sin\theta}$, the third gives $\theta \ge \frac{\pi}{6}$, and the second requires that $r \ge 0$ and $\theta \le \frac{\pi}{2}$ (or $r < 0$ and $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$, but these inequalities result in the same region).