7. [12 points] Each of the following integrals is improper. For each, carefully determine its convergence or divergence by using the comparison test. Be sure to indicate what you are using for your comparison and all steps that allow you to conclude convergence or divergence of the given integral. Mathematical precision is important in this problem.

a. [6 points] $\displaystyle\int_1^\infty \frac{3+\sin\phi}{\phi}\,d\phi$

Solution: A good comparison function is $g(\phi) = \frac{1}{\phi}$ (or $\frac{2}{\phi}$), because for large ϕ the integrand will have the same behavior as $\frac{1}{\phi}$. We know that $g(\phi) \le \frac{3+\sin\phi}{\phi}$, because $3+\sin(\phi) \ge 2$. Then we know that $\int_1^\infty g(\phi)\,d\phi$ diverges; thus $\int_1^\infty \frac{3+\sin\phi}{\phi}\,d\phi$ must also diverge.

b. [6 points] $\displaystyle\int_0^1 \frac{dz}{\sqrt{z^3+z}}$

Solution: A good comparison function is $g(z) = \frac{1}{z^{1/2}}$, because for small z the integrand will look like $\frac{1}{z^{1/2}}$ (z^3 is *less than* z for small values of z). We note that $g(z) > \frac{1}{\sqrt{z^3+z}}$, and know that $\int_0^1 \frac{1}{z^{1/2}}\,dz$ converges. Thus $\int_0^1 \frac{1}{\sqrt{z^3+z}}$ must also converge.