8. [12 points] Some values of the continuous, differentiable function $g(x)$ are given in the table below.

x	1	5/4	3/2	7/4	2
$g(x)$	2	3	4	7	10

a. [6 points] Estimate the integral $\int_1^4 \frac{g(\sqrt{t})}{\sqrt{t}}\,dt$ using these data.

Solution: Let $w = \sqrt{t}$. Then $dw = \frac{1}{2\sqrt{t}}\,dt = \frac{1}{2w}\,dt$, and the integral is $\int_1^4 g(\sqrt{t}\,)\,dt = \int_1^2 2\,g(w)\,dw$. We can estimate this integral from the given data: a left-hand sum is

$$\text{LHS} = \tfrac{1}{4}(2)\,(2+3+4+7) = 8.$$

We could also use a right-hand sum or trapezoid estimate:

$$\text{RHS} = \tfrac{1}{4}\,(2)\ (3+4+7+10) = 12,$$
$$\text{and} \quad \text{TRAP} = \tfrac{1}{2}\,(8+12) = 10.$$

b. [6 points] Estimate the integral $\int_1^4 g(\sqrt{t}\,)\,dt$ using these data.

Solution: Let $w = \sqrt{t}$. Then $dw = \frac{1}{2\sqrt{t}}\,dt = \frac{1}{2w}\,dt$, and the integral is $\int_1^4 g(\sqrt{t}\,)\,dt = \int_1^2 2w\,g(w)\,dw$. We can estimate this integral from the given data:

w	1	5/4	3/2	7/4	2
$g(w)$	2	3	4	7	10
$2w\,g(w)$	4	15/2	12	49/2	40

Thus, a left-hand sum gives $\int_1^2 2w\,g(w)\,dw \approx \frac{1}{4}(4+15/2+12+49/2) = 12$, a right-hand sum gives $\int_1^2 2w\,g(w)\,dw \approx \frac{1}{4}(15/2+12+49/2+40) = 21$, and an average of the two may be expected to be a reasonable estimate for the actual value of the integral:

$$\int_1^2 2w\,g(w)\,dw \approx \frac{33}{2} = 16.5.$$