9. [12 points] For the following, $f(x) > g(x) > 0$ and a is a positive constant. Indicate if each is true or false by circling **True** or **False**. For each, include **one sentence** to explain your answer.

a. [3 points] If $\int_1^\infty f(x)\,dx$ converges, then $\int_1^\infty f(a+x)\,dx$ must converge.

$\boxed{\text{True}}$ False

Solution: $\int_1^\infty f(a+x)\,dx = \int_{1+a}^\infty f(x)\,dx$, which integrates over a smaller region than the convergent integral $\int_1^\infty f(x)\,dx$.

b. [3 points] If $\int_1^\infty f(x)\,dx$ converges, then $\int_1^\infty (a+f(x))\,dx$ must converge.

True $\boxed{\text{False}}$

Solution: $a + f(x) > a$ for all x, and $\int_1^\infty a\,dx$ clearly diverges, so $\int_1^\infty (a+f(x))\,dx$ also diverges.

c. [3 points] If $\int_1^\infty f(x)\,dx$ and $\int_1^\infty g(x)\,dx$ both converge, then $\int_1^\infty f(x)\cdot g(x)\,dx$ must converge.

$\boxed{\text{True}}$ False

Solution: This turns out to be more interesting than one might expect. A reasonable expectation is that $f(x) \to 0$ and $g(x) \to 0$ as $x \to \infty$, so that for large enough x, $f(x)\cdot g(x) < g(x)$ and so by comparison $\int_1^\infty f(x)\cdot g(x)\,dx$ must converge. However, if $f(x)$ and $g(x)$ do not go to zero this may be false. Therefore either True or False was accepted as a correct answer, with credit given for the explanation if it supported the answer given.

d. [3 points] If $\int_1^\infty f(x)\,dx$ and $\int_1^\infty g(x)\,dx$ both converge, then $\int_1^\infty \frac{f(x)}{g(x)}\,dx$ must converge.

True $\boxed{\text{False}}$

Solution: Because $0 < g(x) < f(x)$, $\frac{f(x)}{g(x)} > 1$, so this clearly diverges.