9. [12 points] For the following, \( f(x) > g(x) > 0 \) and \( a \) is a positive constant. Indicate if each is true or false by circling **True** or **False**. For each, include one sentence to explain your answer.

a. [3 points] If \( \int_1^\infty f(x) \, dx \) converges, then \( \int_1^\infty f(a + x) \, dx \) must converge.

**Solution:** \( \int_1^\infty f(a + x) \, dx = \int_{1+a}^\infty f(x) \, dx \), which integrates over a smaller region than the convergent integral \( \int_1^\infty f(x) \, dx \).

b. [3 points] If \( \int_1^\infty f(x) \, dx \) converges, then \( \int_1^\infty (a + f(x)) \, dx \) must converge.

**Solution:** \( a + f(x) > a \) for all \( x \), and \( \int_1^\infty a \, dx \) clearly diverges, so \( \int_1^\infty (a + f(x)) \, dx \) also diverges.

c. [3 points] If \( \int_1^\infty f(x) \, dx \) and \( \int_1^\infty g(x) \, dx \) both converge, then \( \int_1^\infty f(x) \cdot g(x) \, dx \) must converge.

**Solution:** This turns out to be more interesting than one might expect. A reasonable expectation is that \( f(x) \to 0 \) and \( g(x) \to 0 \) as \( x \to \infty \), so that for large enough \( x \), \( f(x) \cdot g(x) < g(x) \) and so by comparison \( \int_1^\infty f(x) \cdot g(x) \, dx \) must converge. However, if \( f(x) \) and \( g(x) \) do not go to zero this may be false. Therefore either True or False was accepted as a correct answer, with credit given for the explanation if it supported the answer given.

d. [3 points] If \( \int_1^\infty f(x) \, dx \) and \( \int_1^\infty g(x) \, dx \) both converge, then \( \int_1^\infty \frac{f(x)}{g(x)} \, dx \) must converge.

**Solution:** Because \( 0 < g(x) < f(x) \), \( \frac{f(x)}{g(x)} > 1 \), so this clearly diverges.