

7. [11 points] A land surveyor is hired to measure the area of a plot of land to be sold. The surveyor uses two main highways as points of reference while measuring the property. Highway 116 is south of the property and runs perfectly in the east-west direction. Highway 1 is west of the property and runs perfectly in the north-south direction. The surveyor starts at Highway 1 and moves eastward for the entire four-mile width of the property as he measures the distances of the northern and southern borders of the property from Highway 116. Let  $n(x)$  and  $s(x)$  be the distances, in miles, of the northern border and southern borders, respectively, from Highway 116 when he is  $x$  miles east of Highway 1. The surveyor's measurements are recorded in the table below.

$x$	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$n(x)$	13.6	13.5	12.9	12.7	12.4	12.0	11.4	11.2	10.9
$s(x)$	7.8	8.1	8.2	8.5	8.7	8.8	9.1	10.0	10.9

- a. [4 points] Estimate the area of the property using the midpoint rule with four subintervals. Be sure to show all appropriate work and don't forget to include appropriate units.
- b. [4 points] Estimate the area of the property using the trapezoid rule with four subintervals. Be sure to show all appropriate work and don't forget to include appropriate units.
- c. [3 points] Because he took calculus, the surveyor knows that he can determine  $\Delta x$ , the uniform distance at which he should make measurements in order to ensure the measured area is within a desired level of accuracy. Given that  $n(x)$  is a decreasing function and  $s(x)$  is an increasing function, determine the value of  $\Delta x$  the surveyor should use in order to measure the area within 0.5 square miles if he is using left- and right-hand Riemann sum approximations for the area.