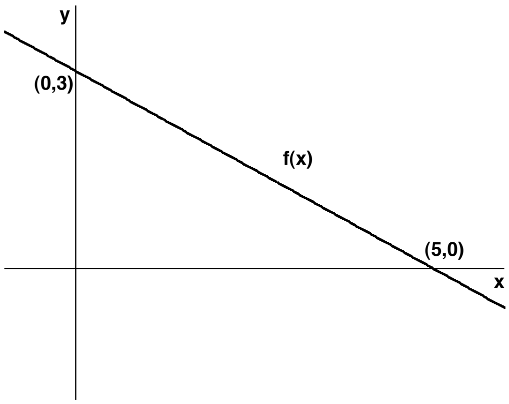
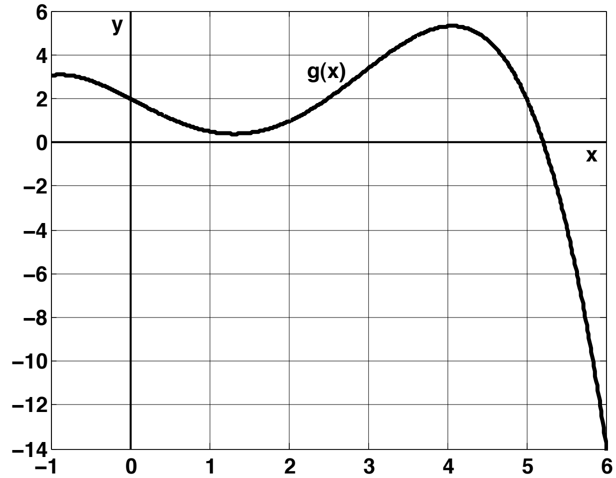
4. [8 points] The graphs of $f(x)$ and $g(x)$ are shown below. Suppose that $f(x)$ is a linear function. Estimate $\int_0^5 f(x)g'(x)dx$. Be sure to show appropriate work to support how you derived your answer.





Solution: We can use integration by parts, letting $u = f(x), du = f'(x)dx, dv = g'(x)dx$, and $v = g(x)$. We then have

$$\int_0^5 f(x)g'(x)dx = f(x)g(x)|_0^5 - \int_0^5 f'(x)g(x)dx.$$

We know that $f(x)$ is a linear function, so from the graph we determine $f(x) = -\frac{3}{5}x + 3$ and $f'(x) = -\frac{3}{5}$. We also know $f(5) = 0, f(0) = 3, g(5) = 2$, and $g(0) = 2$. We can use this to solve in our expression above.

$$\int_0^5 f(x)g'(x)dx = f(5)g(5) - f(0)g(0) + \frac{3}{5}\int_0^5 g(x)dx = -6 + \frac{3}{5}\int_0^5 g(x)dx.$$

By counting boxes, we can approximate $\int_0^5 g(x)dx$, noting that each box has an area of 2. We approximate $\int_0^5 g(x)dx \approx 13$, and so we are left with

$$\int_0^5 f(x)g'(x)dx \approx -6 + \frac{3}{5}(13) = 1.8.$$