6. [10 points] The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Consider the curve described by $y = \sqrt{3x^2 - 3}$, over the domain $2 \leq x \leq 4$. What is the average distance of the points on this curve to the point $(2, 0)$?

Solution: A point on the curve has coordinates $(x, \sqrt{3x^2 - 3})$, so the distance from an arbitrary point on the curve to the point $(2, 0)$ is given by

$$\begin{aligned} D &= \sqrt{(x-2)^2 + (\sqrt{3x^2-3} - 0)^2} \\ &= \sqrt{x^2 - 4x + 4 + 3x^2 - 3} \\ &= \sqrt{4x^2 - 4x + 1} \\ &= \sqrt{(2x-1)^2} \\ &= 2x - 1 \end{aligned}$$

We can use a definite integral to find the average distance over the domain $2 \leq x \leq 4$.

$$\begin{aligned} \text{avg. distance} &= \frac{1}{4-2} \int_2^4 (2x-1)dx \\ &= \frac{1}{2}(x^2 - x)|_2^4 \\ &= \frac{1}{2}(12 - 2) \\ &= 5 \end{aligned}$$

The average distance between a point on the curve $y = \sqrt{3x^2 - 3}$ over the domain $2 \leq x \leq 4$ is 5.