7. [11 points] A land surveyor is hired to measure the area of a plot of land to be sold. The surveyor uses two main highways as points of reference while measuring the property. Highway 116 is south of the property and runs perfectly in the east-west direction. Highway 1 is west of the property and runs perfectly in the north-south direction. The surveyor starts at Highway 1 and moves eastward for the entire four-mile width of the property as he measures the distances of the northern and southern borders of the property from Highway 116. Let $n(x)$ and $s(x)$ be the distances, in miles, of the northern border and southern borders, respectively, from Highway 116 when he is x miles east of Highway 1. The surveyor's measurements are recorded in the table below.

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$n(x)$	13.6	13.5	12.9	12.7	12.4	12.0	11.4	11.2	10.9
$s(x)$	7.8	8.1	8.2	8.5	8.7	8.8	9.1	10.0	10.9

a. [4 points] Estimate the area of the property using the midpoint rule with four subintervals. Be sure to show all appropriate work and don't forget to include appropriate units.

Solution: Using four subintervals, we have $\Delta x = 1$. To determine the length of each subinterval, we consider the distance $n(x) - s(x)$. If we use the midpoint rule to approximate the area, we then have

$$\begin{aligned} \text{MID}(4) &= 1[(n(0.5) - s(0.5)) + (n(1.5) - s(1.5)) + (n(2.5) - s(2.5)) + (n(3.5) - s(3.5))] \\ &= 1(5.4 + 4.2 + 3.2 + 1.2) = 14 \end{aligned}$$

Using the midpoint rule with four subintervals, we find that the area of the land is approximately 14 square miles.

b. [4 points] Estimate the area of the property using the trapezoid rule with four subintervals. Be sure to show all appropriate work and don't forget to include appropriate units.

Solution: Again, we have $\Delta x = 1$ and the lengths are determined by the distance $n(x) - s(x)$. The trapezoid rule uses the average of the left-hand and right-hand sums.

$$\text{RIGHT}(4) = 1[4.7 + 3.7 + 2.3 + 0] = 10.7 \qquad \text{LEFT}(4)1[5.8 + 4.7 + 3.7 + 2.3] = 16.5$$

$$\text{TRAP}(4) = \frac{10.7 + 16.5}{2} = 13.6$$

Using the trapezoid rule with four subintervals, we find that the area of the land is approximately 13.6 square miles.

c. [3 points] Because he took calculus, the surveyor knows that he can determine Δx, the uniform distance at which he should make measurements in order to ensure the measured area is within a desired level of accuracy. Given that $n(x)$ is a decreasing function and $s(x)$ is an increasing function, determine the value of Δx the surveyor should use in order to measure the area within 0.5 square miles if he is using left- and right-hand Riemann sum approximations for the area.

Solution: We solve for Δx under the condition $\Delta x\,|[n(4) - s(4)] - [n(0) - s(0)]| \leq 0.5$, which gives $\Delta x \leq 0.0862$ miles.