

1. [12 points] Indicate if each of the following is true or false by circling the correct answer (Justify your answer):

- a. [3 points] If $F(t)$ and $G(t)$ are antiderivatives of the function $f(t)$ with $F(0) = 1$ and $G(0) = 3$ then $F(2) - G(2) = 1$.

True

 False

$$\boxed{\text{Solution: } F(2) - G(2) = F(0) - G(0) = 1 - 3 = -2}$$

- b. [3 points] If $h(t) > 0$ for $0 \leq t \leq 1$, then the function $H(x) = \int_0^x h(t) dt$ is concave up for $0 \leq x \leq 1$.

True

 False

$$\boxed{\text{Solution: } H'(x) = h(x) > 0, \text{ hence } H(x) \text{ is increasing in } 0 \leq x \leq 1, \text{ but not necessarily concave up. You need } H''(x) = h'(x) > 0 \text{ for } H(x) \text{ to be concave up.}}$$

- c. [3 points] If $\int_0^2 g(t) dt = 6$ then $\int_2^3 3g(2t - 4) dt = 9$.

 True

False

$$\boxed{\text{Solution: Using } u = 2t - 4 \text{ then } \int_2^3 3g(2t - 4) dt = \frac{3}{2} \int_0^2 g(u) du = 9}$$

- d. [3 points] $\frac{d}{dx} \left(\int_{-x^2}^{\sin x} e^{t^3} dt \right) = \cos x e^{x^3} + 2xe^{x^3}$.

True

 False

$$\boxed{\text{Solution: } \frac{d}{dx} \left(\int_{-x^2}^{\sin x} e^{t^3} dt \right) = \cos x e^{\sin^3 x} + 2xe^{-x^6}}$$