**3**. [12 points]

**a**. [8 points] Estimate the value of $I$, where

$$I = \int_0^1 e^{-\frac{t^2}{2}}\,dt$$

using $LEFT(3)$, $RIGHT(3)$, $MID(3)$ and $TRAP(3)$. Write each sum.

*Solution:*

| t | 0 | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{5}{6}$ | 1 |
|---|---|---|---|---|---|---|---|
| $e^{-\frac{t^2}{2}}$ | 1 | .9862 | .946 | .8825 | .8007 | .7006 | .6065 |

$LEFT(3) = \frac{1}{3}(1 + .946 + .8007) = .9156$

$RIGHT(3) = \frac{1}{3}(.946 + .8007 + .6065) = .7844$

$TRAP(3) = \frac{.9156+.7844}{2} = .85$

$MID(3) = \frac{1}{3}(.9862 + .8825 + .7006) = .8564.$

**b**. [4 points] Which among the four Riemann sums ($LEFT(n)$, $RIGHT(n)$, $MID(n)$ and $TRAP(n)$) yields the closest underestimate to $I$, for any number $n$ of subdivisions of the interval $[0, 1]$? Justify your answer.

*Solution:* $f(t) = e^{-\frac{t^2}{2}}$

$f'(t) = -te^{-\frac{t^2}{2}} < 0$ on $t \in (0, 1]$

$f''(t) = e^{-\frac{t^2}{2}}(t^2 - 1) \leq 0$ on $t \in [0, 1]$.

Since $f(t)$ is concave down and decreasing in $[0, 1)$, then $TRAP(n)$ yields the closest underestimate for $I$ for all n.