4. [13 points]

a. [6 points] Compute

$$\int_0^1 f'(x)\sin(2\pi x)dx$$

where $f(x)$ is given by the following table.

x	0	.25	.5	.75	1
$f(x)$	.8	.5	1.2	1	.4

Solution:

$$\int_0^1 f'(x)\sin(2\pi x)dx = f(x)\sin(2\pi x)\,\Big|_0^1 - 2\pi\int_0^1 f(x)\cos(2\pi x)dx$$
$$= -2\pi\int_0^1 f(x)\cos(2\pi x)dx$$

x	0	.25	.5	.75	1
$f(x)\cos(2\pi x)$	.8	0	-1.2	0	.4

$$\text{left sum} = .6283$$
$$\text{right sum} = 1.2566$$
$$\text{trapezoid sum} = .9424$$

b. [7 points] Find

$$\int x^3\cos x^2 dx.$$

Solution:

$$\int x^3\cos x^2 dx =$$
$$u = x^2 \qquad = \frac{1}{2}\int u\cos u du$$
$$\text{by parts} \qquad = \frac{1}{2}\left(u\sin u - \int \sin u du\right)$$
$$= \frac{1}{2}\left(u\sin u + \cos u\right) + C$$
$$= \frac{1}{2}\left(x^2\sin x^2 + \cos x^2\right) + C$$