6. [12 points]

a. [8 points] Robotic submersibles are often used to maintain deep sea oil wells, and the recent BP spill inspired Trevion to design his own. His submersible will have the shape of a cube 1.5 meter in length. If the bottom of the submersible is 800 meters below the surface of the ocean, what is the force exerted by the water on each of the submersible's six outer surfaces? (Water density is $1000\frac{kg}{m^3}$).

Solution:
Top surface: $F = (1000\frac{kg}{m^3})(9.8\frac{m}{s^2})(798.5m)(1.5m)^2 = 17,606,925$ Newtons.

Bottom surface: $F = (1000\frac{kg}{m^3})(9.8\frac{m}{s^2})(800m)(1.5m)^2 = 17,640,000$ Newtons.

Side surfaces: $F = \int_{798.5}^{800} 1000(9.8)(1.5)h dh = 7350h^2\,\big|_{798.5}^{800} = 17,623,462.5$ Newtons

h =distance from the level of the sea.

b. [4 points] Let S be the solid generated by rotating the area enclosed by the curves $y = x^2$ and $y = \sqrt{x}$ around the line $x = 1$. Set up, but do not evaluate a definite integral that can be used to compute the volume of S.

Solution:

$$V = \int_0^1 \pi(r_1^2 - r_2^2)dy$$
$$= \int_0^1 \pi((1-y^2)^2 - (1-\sqrt{y})^2)dy$$

where
r_1 = horizontal distance between the line $x = 1$ and the graph $y = \sqrt{x}, (x = y^2)$
r_2 = horizontal distance between the line $x = 1$ and the graph $y = x^2, (x = \sqrt{y})$