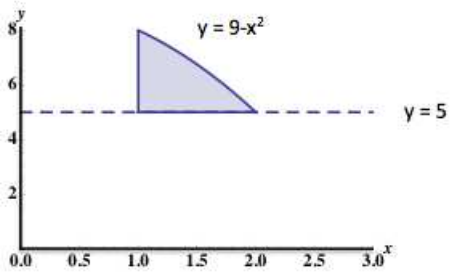
4. [12 points] Consider the region in the xy-plane bounded by the curves $y = 9 - x^2, x = 1$, and $y = 5$. This region is pictured below.



Give a definite integral that computes the quantities below. You do not need to evaluate these integrals.

a. [3 points] The area of the region shown.

Solution:
A= $\int_1^2 (9 - x^2) - 5dx = \int_1^2 4 - x^2 dx$.

A= $\int_5^8 \sqrt{9 - y} - 1 dy$.

b. [3 points] The volume of the solid obtained by rotating the region about the y-axis.

Solution:

$$V = \int_5^8 \pi \left(\left(\sqrt{9 - y} \right)^2 - 1 \right) dy \quad \text{or} \quad V = \int_1^2 2\pi x \left(9 - x^2 - 5 \right) dx = \int_1^2 2\pi x \left(4 - x^2 \right) dx$$

c. [3 points] The volume of the solid obtained by rotating the region about the x-axis.

Solution:

$$V = \int_1^2 \pi \left(\left(9 - x^2 \right)^2 - 25 \right) dx \quad \text{or} \quad V = \int_5^8 2\pi y \left(\sqrt{9 - y} - 1 \right) dy$$

d. [3 points] The volume of the solid obtained by rotating the region about the line $y = 5$.

Solution:

$$V = \int_1^2 \pi \left(4 - x^2 \right)^2 dx \quad \text{or} \quad V = \int_5^8 2\pi (y - 5) \left(\sqrt{9 - y} - 1 \right) dy$$