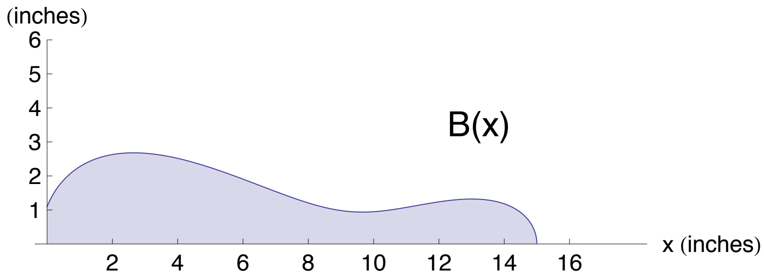
5. [11 points] During a friendly game of ten-pin bowling, your friends Walter and Smokey begin to quarrel over whether Smokey's toe slipped over the foul line. Meanwhile, you decide to pass the time by finding a mathematical model for the shape of a bowling pin. After some careful thought, you find that a fallen pin is a solid of revolution given by rotating the region under the curve

$$B(x) = \sqrt{1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4}$$

over the interval $[0, 15]$ about the x-axis. The region is pictured below. All measurements are in inches. A helpful stranger in the bowling alley informs you that the wood used to make the pin has density $\delta = 17$ grams per cubic inch.



a. [3 points] Write a definite integral that gives the mass of the bowling pin. You do not need to evaluate this integral.

Solution: Since the bowling pin is a solid of revolution, the volume of a cylindrical slice located x inches from the base of the pin can be approximated by $\pi B(x)^2 \Delta x$. Thus, the mass of the slice is approximately

$$\delta \pi B(x)^2 \Delta x = 17\pi \left(\sqrt{1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4}\right)^2 \Delta x,$$

so that the mass of the entire pin is

$$\int_0^{15} 17\pi B(x)^2 dx = \int_0^{15} 17\pi \left(1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4\right) dx \text{ grams.}$$

b. [6 points] What are the coordinates $(\bar{x}, \bar{y})$ of the bowling pin's center of mass? You may use your calculator to answer this question.

Solution: Since the bowling pin has uniform density, we know immediately from rotational symmetry that $\bar{y} = 0$. Using the formula for the x-coordinate of the center of mass, we have

$$\begin{aligned}\bar{x} &= \frac{\int_0^{15} \delta x \pi B(x)^2 dx}{\int_0^{15} \delta \pi B(x)^2 dx} \\ &= \frac{\int_0^{15} 17x\pi \left(1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4\right) dx}{\int_0^{15} 17\pi \left(1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4\right) dx} = 5.\end{aligned}$$

Thus, the coordinates of the center of mass of the bowling pin are $(5, 0)$, where each coordinate is measured in inches.

c. [2 points] Suppose the wood used to make the pin had density $\delta = 16$ grams per cubic inch. How does this affect the position $(\bar{x}, \bar{y})$ of the center of mass?

Solution: The center of mass is not affected since the integral

$$\begin{aligned}\bar{x} &= \frac{\int_0^{15} \delta x \pi B(x)^2 dx}{\int_0^{15} \delta \pi B(x)^2 dx} \\ &= \frac{\int_0^{15} x \pi B(x)^2 dx}{\int_0^{15} \pi B(x)^2 dx}\end{aligned}$$

is independent of δ. The same is true for $\bar{y}$.