6. [14 points] A botanical garden has the shape of the region in the xy-plane bounded by the curve $y = x^2$ and the x-axis, with $0 \leq x \leq 8$. One of the responsibilities of the gardener, is to keep the garden free of a poisonous weed. The density δ of the weed at any point in the garden depends on the distance x from the y-axis. Values of δ are given in kg of plants per meter square in the table below.

x (meters)	0	2	4	6	8
$\delta(x)$	10	12	15	17	18

a. [3 points] Write an integral that computes the total amount of weed in the garden. Include units.

Solution: $\int_0^8 x^2 \delta(x) dx$ kg .

b. [3 points] Compute RIGHT(4) for the integral in (a). Write out all the terms in the sum. Does this sum give an overestimate or an underestimate for the total amount of weed in the garden? Justify.

Solution: $\int_0^8 x^2 \delta(x) dx \approx 2(2^2 \cdot 12 + 4^2 \cdot 15 + 6^2 \cdot 17 + 8^2 \cdot 18) = 4104$.

RIGHT(4) is an overestimate because $\delta(x)$ is increasing and x^2 is increasing then $x^2 \delta(x)$ is increasing.

c. [2 points] Which of the following approximations to (a) are computable with the given data? Circle all that apply.

$$\text{MID}(1) \qquad \text{MID}(2) \qquad \text{MID}(3) \qquad \text{MID}(4)$$

Solution:

$$\underline{\text{MID}(1)} \qquad \underline{\text{MID}(2)} \qquad \text{MID}(3) \qquad \text{MID}(4)$$

d. [1 point] Which Riemann sum gives the best estimate for the integral in (a)? Circle one.

$$\text{RIGHT}(4) \qquad \text{LEFT}(4) \qquad \text{TRAP}(4)$$

Solution:

$$\text{RIGHT}(4) \qquad \text{LEFT}(4) \qquad \underline{\text{TRAP}(4)}$$

e. [5 points] The gardener built a fence around the garden. How long is the fence? Include units. You may use your calculator.

Length of the fence $= 8 + 64 + \int_0^8 \sqrt{1 + \left(\frac{d}{dx}(x^2)\right)^2} dx = 72 + \int_0^8 \sqrt{1 + 4x^2} dx = 136.99$ meters.