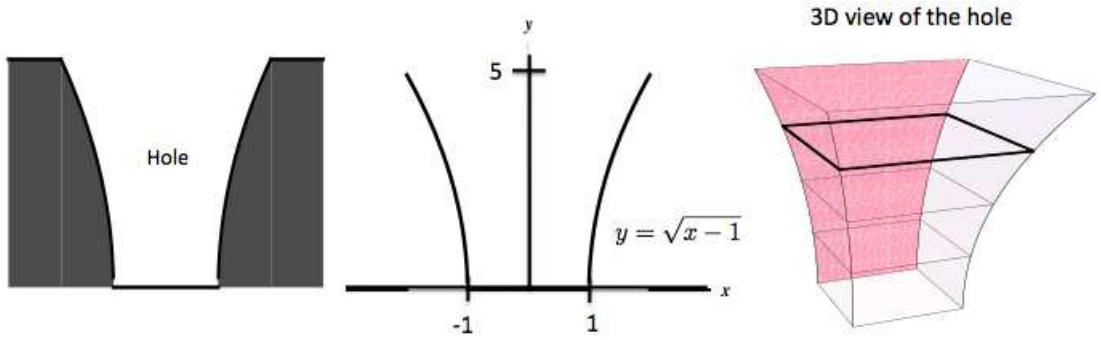
**9**. [9 points] As part of an exploration assignment, a team of mining engineers dug a hole in the ground. The hole takes the shape of a solid region of known cross-section. The base region, which stands vertically, is pictured below. Cross-sections taken perpendicular to the $y$-axis are squares with one side lying on the $xy$-plane.




The variables $x$ and $y$ are given in meters.

**a**. [6 points] Take a slice of soil of thickness $\Delta y$ meters located at $y$ meters above bottom of the hole. Write an expression that approximates the amount of work necessary to move that slice of soil to the top of the hole. The density of the soil is 1600 $\frac{\text{kg}}{\text{m}^3}$. Show all work to receive full credit.

*Solution:*

$$\begin{aligned} W_{slice} &= F_{slice}\ d_{slice} = (\delta_{soil}\ V_{slice}\ g)\ d_{slice} \\ &\approx 1600(2(1+y^2))^2 \Delta y (9.8)(5-y) \\ &= 62720(1+y^2)^2(5-y)\Delta y \end{aligned}$$

**b**. [3 points] How much work does it take to dig the hole? You may use your calculator to answer this question. Include units.

*Solution:*

$$W_{hole} = \int_0^5 62720(1+y^2)^2(5-y)dy = 39,984,000 \text{ Joules.}$$