7. [10 points] Consider the solid \( S \) whose base is the region bounded by the circle \( x^2 + y^2 = 4 \) and the \( y \)-axis with \( 0 \leq x \leq 2 \) in the \( xy \)-plane, and whose cross-sections perpendicular to the \( x \)-axis are half ellipses. The major and minor axes of the ellipses satisfy \( a = \frac{1}{4}b \) (see the picture below). The \( x \) and \( y \) are measured in centimeters.

The area of an ellipse is \( A = \pi ab \).

a. [6 points] Write a definite integral that computes the volume of the solid \( S \). You do not need to evaluate the integral. Include units.

b. [4 points] The mass density of \( S \) is \( \delta(x) = 4 + x^2 \) mg per cm\(^3\). Find the mass of \( S \). You may use your calculator to evaluate any integrals. Include units.