1. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] Let $u(x)$ and $v(x)$ be differentiable functions with $u(0) = u(1) = 0$, then

$$\int_0^1 u(x)v'(x)dx = -\int_0^1 u'(x)v(x)dx.$$

$\boxed{\text{True}}$ False

Solution: Using integration by parts

$$\int_0^1 u(x)v'(x)dx = -u(x)v(x)\,|_0^1 - \int_0^1 u'(x)v(x)dx = -\int_0^1 u'(x)v(x)dx.$$

b. [2 points] The function $f(x) = \displaystyle\int_0^{x^2} e^{t^2} dt$ is decreasing for $x < 0$.

$\boxed{\text{True}}$ False

Solution: $f'(x) = 2xe^{x^4} < 0$ for $x < 0$, hence $f(x)$ is decreasing for $x < 0$.

c. [2 points] For any differentiable function $f(x)$

$$\int_0^x f'(t)dt = \frac{d}{dx}\left(\int_0^x f(t)dt\right).$$

True $\boxed{\text{False}}$

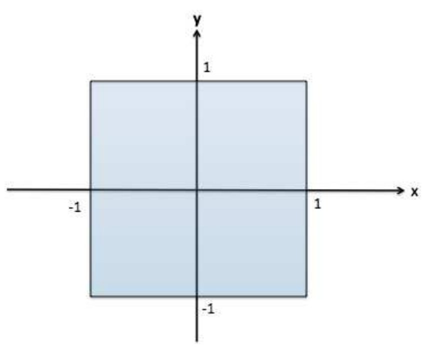
Solution:

$$\int_0^x f'(t)dt = f(x) - f(0) \text{ by the Fundamental Theorem of Calculus}$$

$$\frac{d}{dx}\left(\int_0^x f(t)dt\right) = f(x) \text{ by the Second Fundamental Theorem of Calculus}$$

Hence it is not true for functions for which $f(0) \neq 0$, example $f(x) = x + 1$.

d. [2 points] If the mass density function of a square plate (shown below) is $\delta(y)$, an even function of y only, then the center of mass of the plate lies on the x-axis.



$\boxed{\text{True}}$ False

Solution: The y-coordinate of the center of mass is given by

$$\bar{y} = \frac{\int_{-1}^{1} 2\delta(y) y dy}{\int_{-1}^{1} 2\delta(y) dy} = 0.$$

Since the integrand $2\delta(y)y$ is an odd function of y, then $\displaystyle\int_{-1}^{1} 2\delta(y) y dy = 0$.

e. [2 points] If we use the trapezoidal rule to approximate the integral $I = \int_0^1 (1+2t)dt$ then Trap(n) is exactly equal to I for every n.

$\boxed{\text{True}}$ False

Solution: The area represented by $\int_0^1 (1+2t)dt$ is a trapezoid, hence the trapezoid rule gives you the exact value of this area.

f. [2 points] If $f(x)$ is concave up, then the average value of $f(x)$ on the integral $[0, 2]$ is larger than $f(1)$.

$\boxed{\text{True}}$ False

Solution: If A is the average value of $f(x)$ on the interval $[0, 2]$, then $2A = \int_0^2 f(x)dx$. We know that if $f(x)$ is concave up, then $Mid(1) = 2f(1) < \int_0^2 f(x)dx = 2A$. Hence $f(1) < A$.