4. [16 points] Consider the region R bounded by the graphs of $y = \ln(x)$, $y = 0$ and $x = 2$. In the following questions, show all your work to receive full credit.

a. [4 points] Find the perimeter of the region R. You may use your calculator to evaluate any integrals.

Solution: $L = 1 + \ln 2 + \int_1^2 \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx \approx 2.915.$

b. [5 points] Let S be the solid obtained by rotating the region R about the y axis. Write an expression for the volume of a slice of the solid S located at a height y with thickness Δy.

Solution: $V_{slice} \approx \pi[4 - e^{2y}]\Delta y.$

c. [2 points] Suppose S has mass density $\delta(y) = e^{-y}$. Write an expression for the mass of the solid S using a definite integral. You do not need to evaluate this integral.

Solution: $\text{Mass} = \int_0^{\ln 2} e^{-y}\pi[4 - e^{2y}]dy.$

d. [2 points] What is the value of $\bar{x}$, the x coordinate of the center of mass of S? Justify.

Solution: S is a solid of revolution where the y axis is its axis of symmetry and the mass density $\delta(y)$ is independent of x, hence the center of mass should be on the y axis. Hence $\bar{x} = 0$.

e. [3 points] Write an expression for $\bar{y}$, the y coordinate of the center of mass of S, using definite integrals. You do not need to evaluate this expression.

Solution:

$$\bar{y} = \frac{\int_0^{\ln 2} y e^{-y}\pi[4 - e^{2y}]dy}{\int_0^{\ln 2} e^{-y}\pi[4 - e^{2y}]dy}.$$