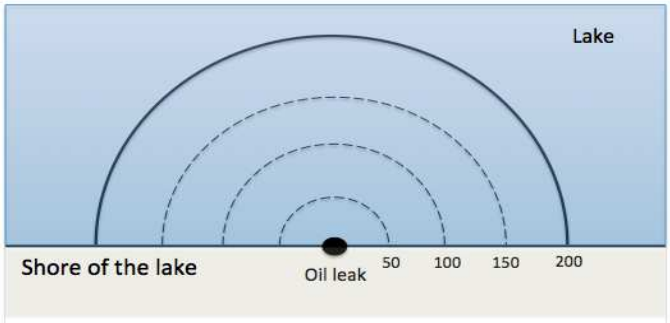
5. [12 points] Oil leaks from a tank on the shore of a lake to form a semicircular slick on the surface of the water (as shown in the figure below). A team of environmentalists is trying to estimate the amount of oil spilled. They took measurements of the density P of oil (in kg per m^2) in the slick and found that it was a function of the distance r (in m) from the source of the oil.



The values of $P(r)$ measured by the environmentalists are shown in the table below.

r	0	50	100	150	200
$P(r)$	100	40	12	10	8

a. [6 points] Write an expression involving integrals for the exact value of the mass of the oil in the lake inside a semicircle centered at the oil leak with a radius of 200 meters (see the figure above). Include units.

Solution: $\text{Mass}_{oil} = \displaystyle\int_0^{200} \pi r P(r)\,dr$ kg.

b. [4 points] Find approximations to your answer in part (**a**) using Left(4) and Right(4). Show your work by writing all the terms of the sums.

Solution:

r	0	50	100	150	200
$P(r)$	100	40	12	10	8
$\pi r P(r)$	0	6,283.2	3,769.9	4,712.4	5,026.5

$$\text{Left}(4) = 50(6,283.2 + 3,769.9 + 4,712.4) = 738,274.27 \text{ kg}$$
$$\text{Right}(4) = 50(6,283.2 + 3,769.9 + 4,712.4 + 5,026.5) = 989,601.68 \text{ kg}$$

c. [2 points] The environmentalists notice that the density $P(r)$ of oil is a decreasing function. Does this observation guarantee that one of the approximations in part (**b**) yields an overestimate? If so, which one? Justify.

Solution: To get an overestimate, the function $Q(r) = \pi r P(r)$ needs to be either increasing or decreasing for $0 \le r \le 200$. The fact that $P(r)$ is decreasing it is not enough. In this case, $Q(r)$ is neither increasing or decreasing (look at its values at $r = 0, 50, 100$).