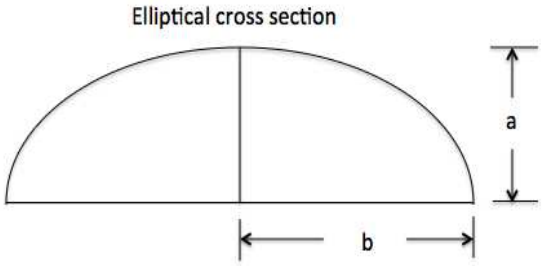
7. [10 points] Consider the solid S whose base is the region bounded by the circle $x^2 + y^2 = 4$ and the y-axis with $0 \leq x \leq 2$ in the xy-plane, and whose cross-sections perpendicular to the x-axis are half ellipses. The major and minor axes of the ellipses satisfy $a = \frac{1}{4}b$ (see the picture below). The x and y are measured in centimeters.



The area of an ellipse is $A = \pi ab$.

a. [6 points] Write a definite integral that computes the volume of the solid S. You do not need to evaluate the integral. Include units.

Solution:

$$V_{slice} \approx A_{slice}\ \Delta x = \frac{1}{2}\pi ab\ \Delta x = \frac{1}{2}\pi \left(\frac{1}{4}\sqrt{4-x^2}\right)\left(\sqrt{4-x^2}\right)\Delta x$$

$$V_{slice} \approx \frac{1}{8}\pi(4-x^2)\Delta x.$$

$$V_{solid} = \int_0^2 \frac{1}{8}\pi(4-x^2)dx \text{ cm}^3$$

b. [4 points] The mass density of S is $\delta(x) = 4 + x^2$ mg per cm^3. Find the mass of S. You may use your calculator to evaluate any integrals. Include units.

Solution:

$$\text{Mass}_{solid} = \int_0^2 \frac{1}{8}\pi(4-x^2)(4+x^2)dx \approx 10.053 \text{ mg}$$