3. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] Let \( F(x) \) be an antiderivative of a function \( f(x) \). Then \( F(2x) \) is an antiderivative of the function \( f(2x) \).

\[
\begin{array}{ll}
\text{True} & \text{False} \\
\end{array}
\]

Solution: Let \( f(x) = 3x^2 \), then an antiderivative is \( F(x) = x^3 \) since \( F'(x) = 3x^2 = f(x) \), but \( F(2x) = (2x)^3 = 8x^3 \) is not an antiderivative of \( f(2x) = 3(2x)^2 = 12x^2 \) since \( \frac{d}{dx} F(2x) = \frac{d}{dx} (8x^3) = 24x^2 \neq f(2x) = 12x^2 \).

b. [2 points] If \( f(x) \) is a linear function on \([0, 1]\), then the midpoint rule computes the exact value of \( \int_0^1 f(x)\,dx \).

\[
\begin{array}{ll}
\text{True} & \text{False} \\
\end{array}
\]

c. [2 points] If \( f(x) \) is a negative function that satisfies \( f'(x) > 0 \) for \( 0 \leq x \leq 1 \). Then the right hand sums always yield an underestimate for the value of \( \int_0^1 (f(x))^2\,dx \).

\[
\begin{array}{ll}
\text{True} & \text{False} \\
\end{array}
\]

Solution: Let \( g(x) = f(x)^2 \), then \( g'(x) = 2f(x)f'(x) < 0 \) on \([0, 1]\). Since \( g(x) \) is decreasing, then the right hand sum yields an underestimate for \( \int_0^1 g(x)\,dx = \int_0^1 (f(x))^2\,dx \).

d. [2 points] If \( a \) and \( b \) are positive constants, then \( \int e^{ax+b}dx = \frac{1}{2ax} e^{ax+b} + C \).

\[
\begin{array}{ll}
\text{True} & \text{False} \\
\end{array}
\]

Solution: Since \( \frac{d}{dx} \left( \frac{1}{2ax} e^{ax+b} \right) \neq e^{ax+b} \), then the formula above is not true.

e. [2 points] The average value of \( f(x)g(x) \) on an interval \([a, b]\) is the average value of \( f(x) \) on \([a, b]\) times the average value of \( g(x) \) on \([a, b]\).

\[
\begin{array}{ll}
\text{True} & \text{False} \\
\end{array}
\]

Solution: Let \( f(x) = x, g(x) = 1-x \) and \([a, b] = [0, 1]\), then \( \int_0^1 f(x)\,dx = \int_0^1 g(x)\,dx = \frac{1}{2} \), but \( \int_0^1 f(x)g(x)\,dx = \int_0^1 x(1-x)\,dx = \frac{1}{6} \neq \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) = \frac{1}{4} \).

f. [2 points] If \( k > 0 \) is a constant, the arclength of the function \( y = kf(x) \) on an interval \([a, b]\) is \( k \) times the arclength of \( y = f(x) \) on \([a, b]\).

\[
\begin{array}{ll}
\text{True} & \text{False} \\
\end{array}
\]

Solution: Let \( f(x) = 1, [a, b] = [0, 1] \) and \( k = 2 \), then the arclength of \( f(x) \) on \([0, 1]\) is 1. The arclength of \( y = 2f(x) \) on \([0, 1]\) is 1, not 2(1) = 2.