3. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] Let $F(x)$ be an antiderivative of a function $f(x)$. Then $F(2x)$ is an antiderivative of the function $f(2x)$.

True **[False]**

Solution: Let $f(x) = 3x^2$, then an antiderivative is $F(x) = x^3$ since $F'(x) = 3x^2 = f(x)$, but $F(2x) = (2x)^3 = 8x^3$ is not an antiderivative of $f(2x) = 3(2x)^2 = 12x^2$ since $\frac{d}{dx}(F(2x)) = \frac{d}{dx}(8x^3) = 24x^2 \neq f(2x) = 12x^2$.

b. [2 points] If $f(x)$ is a linear function on $[0,1]$, then the midpoint rule computes the exact value of $\int_0^1 f(x)dx$.

[True] False

Solution:

c. [2 points] If $f(x)$ is a negative function that satisfies $f'(x) > 0$ for $0 \leq x \leq 1$. Then the right hand sums always yield an underestimate for the value of $\int_0^1 (f(x))^2 dx$.

[True] False

Solution: Let $g(x) = f(x)^2$, then $g'(x) = 2f(x)f'(x) < 0$ on $[0,1]$. Since $g(x)$ is decreasing, then the right hand sum yields an underestimate for $\int_0^1 g(x)dx = \int_0^1 (f(x))^2 dx$

d. [2 points] If a and b are positive constants, then $\int e^{ax^2+b}dx = \frac{1}{2ax}e^{ax^2+b} + C$.

True **[False]**

Solution: Since $\frac{d}{dx}\left(\frac{1}{2ax}e^{ax^2+b}\right) \neq e^{ax^2+b}$, then the formula above is not true.

e. [2 points] The average value of $f(x)g(x)$ on an interval $[a,b]$ is the average value of $f(x)$ on $[a,b]$ times the average value of $g(x)$ on $[a,b]$.

True **[False]**

Solution: Let $f(x) = x$, $g(x) = 1-x$ and $[a,b] = [0,1]$, then $\int_0^1 f(x)dx = \int_0^1 g(x)dx = \frac{1}{2}$, but $\int_0^1 f(x)g(x)dx = \int_0^1 x(1-x)dx = \frac{1}{6} \neq \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$.

f. [2 points] If $k > 0$ is a constant, the arclength of the function $y = kf(x)$ on an interval $[a,b]$ is k times the arclength of $y = f(x)$ on $[a,b]$.

True **[False]**

Solution: Let $f(x) = 1$, $[a,b] = [0,1]$ and $k = 2$, then the arclength of $f(x)$ on $[0,1]$ is 1. The arclength of $y = 2f(x)$ on $[0,1]$ is 1, not $2(1) = 2$.