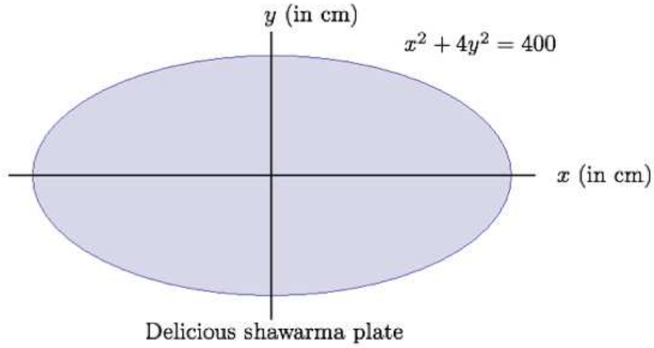
4. [17 points]

a. [8 points] The delicious chicken shawarma platter is served on an elliptical plate, described by the equation $x^2 + 4y^2 = 400$. The mass density of the platter, including the food, is a function of y, given by $\delta(y) = 10 + 0.5y$ grams per cm^2.

In this problem, you do not need to evaluate any integrals.



i) (4 points) Find an expression containing a definite integral that computes the mass of the chicken shawarma platter (including the food).

Solution:

$$m = \int_{-10}^{10} 2\sqrt{400 - 4y^2} \cdot (10 + 0.5y)dy.$$

ii) (4 points) Find expressions for the coordinates $\bar{x}$, $\bar{y}$ of the center of mass of the platter. If your expression does not involve an integral, include a justification.

Solution: $\bar{x} = 0$, since both the shape and density function are symmetric about the y-axis.

$$\bar{y} = \frac{\int_{-10}^{10} y \cdot 2\sqrt{400 - 4y^2} \cdot (10 + 0.5y)dy}{\int_{-10}^{10} 2\sqrt{400 - 4y^2} \cdot (10 + 0.5y)dy}.$$

b. [9 points] The mouthwatering kafta kabob platter is served on a circular plate, with radius 20 cm. Including the food, the overall mass density of the platter is given by $\delta(r) = \dfrac{50}{2+r^2}$ grams per cm^2, where r is the distance from the center of the plate (in cm).

i) (4 points) Write a definite integral that computes the mass of the kafta kabob platter (including food). You do not need to evaluate the integral.

Solution:

$$m = \int_0^{20} 2\pi r \cdot \frac{50}{2+r^2} dr.$$

ii) (3 points) Write an estimate for your expression in part i) of the mass of the platter using LEFT(3). Show all the terms in the sum. You do not need to evaluate the sum.

Solution:

$$\begin{aligned} \text{LEFT}(3) &= \frac{20}{3}\left(2\pi \cdot 0 \cdot \frac{50}{2+0^2} + 2\pi \cdot \frac{20}{3} \cdot \frac{50}{2+(\frac{20}{3})^2} + 2\pi \cdot \frac{40}{3} \cdot \frac{50}{2+(\frac{40}{3})^2} \right) \\ &= \frac{20}{3}\left(0 + 45.09 + 23.30 \right). \\ &= \quad 0 + 300.6 + 155.33. \end{aligned}$$

iii) (2 points) Where is the center of mass of this platter? Justify.

Solution: At the center of the plate, since both the shape and density function are symmetric about the origin.