8. [10 points] Let $g(x)$ be an odd continuous function and $h(x)$ an antiderivative of $g(x)$. Find the values of the following expressions given the values of the functions below.

x	0	1	2	3	4
$g(x)$	0	5	1	-2	-1
$h(x)$	2	6	10	8	5

Show all your work to receive full credit.

a. [3 points] $\displaystyle\int_{-1}^{2}(3g(t)-5)dt$

Solution:

$$\int_{-1}^{2}(3g(t)-5)dt = 3\int_{-1}^{2} g(t)dt - 5\int_{-1}^{2} dt$$

$$\text{since } g(t) \text{ is odd } \int_{-1}^{1} g(t)dt = 0.$$

$$\begin{aligned}3\int_{-1}^{2} g(t)dt - 5\int_{-1}^{2} dt &= 3\int_{1}^{2} g(t)dt - 5(3)\\ &= 3(h(2)-h(1)) - 15\\ &= 3(10-6) - 15 = 12 - 15 = -3.\end{aligned}$$

b. [4 points] $\displaystyle\int_{1}^{3} tg'(t)dt$

Solution: Using integration by parts with $u = t$, $v' = g'(t)$, then $u' = 1$ and$v = g(t)$,

$$\begin{aligned}\int_{1}^{3} tg'(t)dt &= tg(t)\,\Big|_1^3 - \int_{1}^{3} g(t)dt\\ &= 3g(3) - g(1) - (h(3) - h(1))\\ &= 3(-2) - 5 - (8-6) = -6 - 5 - 2 = -13.\end{aligned}$$

c. [3 points] Let $F(x) = \displaystyle\int_{3}^{4x} h(t)dt$. Find $F'(1)$.

Solution: Using the second Fundamental Theorem of Calculus,

$$F'(x) = 4h(4x) \qquad \text{then} \qquad F'(1) = 4h(4) = 20.$$