9. [11 points] Consider the region R bounded by $y = 2(1 - e^{-x})$, $y = 1$ and $x = 3$. In this problem, you do not need to evaluate the integrals.

a. [4 points] Find a definite integral that computes the volume of the solid obtained by rotating the region R about the y-axis.

Solution:

•Washer method:

$$V = \int_1^{2(1-e^{-3})} \pi \left(3^2 - \left(-\ln\left(1 - \frac{y}{2}\right)\right)^2\right) dy$$
$$= \int_1^{2(1-e^{-3})} \pi \left(9 - \ln^2\left(1 - \frac{y}{2}\right)\right) dy$$

•Shell method:

$$V = \int_{\ln 2}^{3} 2\pi x \left(2(1 - e^{-x}) - 1\right) dx$$

b. [7 points] Find a definite integral that computes the volume of the solid with base given by the region R, and whose cross sections perpendicular to the x-axis are semicircles.

Solution:

$$V = \int_{\ln 2}^{3} \frac{1}{2} \left(\pi \left(\frac{1}{2}\left[2(1 - e^{-x}) - 1\right]\right)^2\right) dx$$
$$= \int_{\ln 2}^{3} \frac{\pi}{8} \left[2(1 - e^{-x}) - 1\right]^2 dx$$