1. [13 points] Let $g(x)$ be a differentiable, **odd** function and let $G(x)$ be an anti-derivative of $g(x)$ with $G(2) = 0$. A table of values for $g(x)$ and $G(x)$ is provided below. **Be sure to show all of your work.**

x	0	1	2	3	4
$g(x)$	0	2	3	4	5
$G(x)$	-7	-4	0	5	9

a. [2 points] Write down a formula for $G(x)$ in terms of the function $g(t)$.

$G(x) =$ $\int_2^x g(t)dt$

b. [2 points] Compute $\int_0^1 g(x)dx$.

Solution:

$$\int_0^1 g(x)dx = G(1) - G(0) = -4 - (-7) = 3$$

c. [3 points] Compute $\int_{-4}^2 g(x)dx$.

Solution:

$$\begin{aligned}\int_{-4}^2 g(x)dx &= \int_{-4}^{-2} g(x)dx \\ &= -\int_2^4 g(x)dx \\ &= -(G(4) - G(2)) \\ &= G(2) - G(4) \\ &= -9\end{aligned}$$

d. [3 points] Compute $\int_1^3 xg'(x)dx$.

Solution:

$$\begin{aligned}\int_1^3 xg'(x)dx &= xg(x)|_1^3 - \int_1^3 g(x)dx \\ &= 3g(3) - 1g(1) - (G(3) - G(1)) \\ &= 12 - 2 - (5 - (-4)) \\ &= 1\end{aligned}$$

e. [3 points] Compute $\int_0^1 g(3x)dx$.

Solution:

$$\int_0^1 g(3x)dx = 1/3\int_0^3 g(x)dx = 1/3(G(3) - G(0)) = 1/3(5 - (-7)) = 4$$