4.*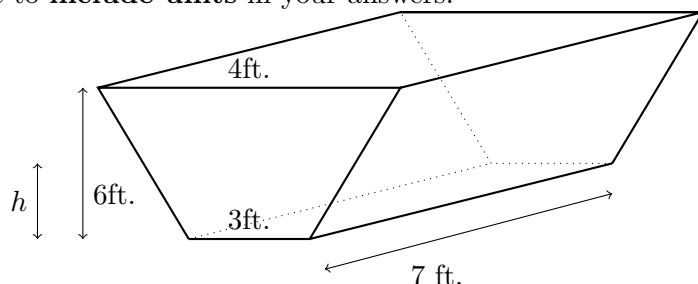* [14 points] Franklin, your robot, has been digging a ditch in the backyard. The ditch is a trapezoidal prism with length 7 feet, width at the bottom 3 feet and width at the top 4 feet. The ditch descends 6 feet underground. You may assume that each cubic foot of dirt weighs 94 pounds. Be sure to **include units in your answers.

a. [5 points] Write an expression that approximates the work that Franklin does lifting a slice of dirt Δh feet thick from h feet above the bottom of the ditch, to the top of the ditch.

Solution:

$$\text{Work to lift the slice} = (\text{Displacement})(\text{Weight})$$
$$= (6-h) \cdot \delta(7)(\text{Width of the slice})\Delta h$$

The width of the slice is given by $w = 3 + h/6$

$$\text{Work to lift the slice} = (6-h)\delta(7(3+h/6))\Delta h \quad \text{foot pounds}$$

b. [3 points] Using your answer to part **(a)**, write an integral that computes the total work required to move all of the dirt out over the top of the ditch.

Solution:

$$\text{Total Work} = \int_0^6 (6-h)94(7(3+h/6))dh$$

c. [3 points] To lift the dirt, Franklin uses your old rusty bucket. For Franklin's last bucket-full of dirt, the bucket starts with 30 pounds of dirt and loses 10 pounds of dirt at a constant rate over the 6 feet that it travels vertically. When the bucket is x feet above the bottom of the ditch, what is the weight of the dirt in the bucket?

Solution: When the bucket is x feet above the bottom of the hole, it contains $30 - \frac{10x}{6}$ pounds of dirt.

d. [3 points] Using your answer from part **(c)**, find the work required to lift the bucket-full of dirt from the bottom of the ditch to the top.

Solution:

$$\text{Total Work} = \int_0^6 (30 - \frac{10x}{6})dx = 150 \quad \text{foot pounds}$$